Trajectory Generation for a Self-Driving Car as a Constrained Optimization Problem

Advanced driver assistance system and self-driving cars which steer autonomously are one of the big recent trends in the automotive industry. Starting from current research projects, the technology is expected to be available in production cars gradually. Promising concepts include highway pilot, automated valet parking and emergency braking systems.
Different approaches exist to plan a trajectory (e.g. convert it to a graph search problem)

One approach is to formulate it as a constrained optimization problem:

$$\min_u J(x(t)) = \Phi(x(t_0 + T)) + \int_{t_0}^{t_0+T} L(x, \dot{x}, \ddot{x}, \ldots ) dt$$

subject to:  
$$\dot{x} = f(x, t, u)$$

$$O(t) \cap x(t) = \emptyset$$

... 

In order to be able to solve this problem, the cost and constraints must be chosen in an appropriate way.

How to choose the costs $L$? Which vehicle model $f(x, t, u)$? How to represent obstacles in the formulations?

Ref.: Ziegler, J. ; Bender, P. ; Thao Dang ; Stiller, C.: Trajectory planning for Bertha A local, continuous method *Intelligent Vehicles Symposium Proceedings*, 2014 IEEE