Real-Time Systems

Part 8: Scheduling
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2. Scheduling Algorithms
   a. Overview
   b. Offline Schedulers
   c. Online Schedulers

3. Schedulability Testing

4. Resources and Resource Access Control
Literature

- Jane W. S. Liu, Real-Time Systems, 2000
- Fridolin Hofmann: Betriebssysteme - Grundkonzepte und Modellvorstellungen, 1991
- Klaus Gresser, Echtzeitnachweis ereignisgesteuerter Realzeitsysteme, Dissertation, TUM, 1993

Journals:


- Giorgio C. Buttazzo: Rate Monotonic vs. EDF: Judgement Day ([http://www.cas.mcmaster.ca/~downd/rtsj05-rmedf.pdf](http://www.cas.mcmaster.ca/~downd/rtsj05-rmedf.pdf))


Introduction
Scheduler and Dispatcher

• **Scheduler:**

If a resource is to be used by many consumers, access to the resource has to be coordinated. This resource allocation is performed by a **scheduler**.

In computer systems, the term scheduler often refers to the CPU scheduler which controls the allocation of the CPU to **tasks**.

• **Dispatcher:**

While the scheduler plans the CPU allocation, the dispatcher executes the scheduler plan by:

- Switching the context
- Switching to user mode
- Jumping to the proper location in the user program to restart it
We introduce the following model for a task:

- **Release Time (or arrival time) \( r_i \)**
  Earliest time at which task \( i \) is enabled.

- **Start Time \( s_i \)**
  Time at which execution of task starts.

- **Finish Time \( f_i \)**
  Time at which task completes execution.

- **Response Time \( O_i \)**
  Interval between release and finish time.
We introduce the following model for a task:

- **Execution Time** $e_i$  
  *(remaining execution time $\hat{e}_i$ – see next slide)*
  Total time of task execution (does not include durations where the task was blocked).

- **Relative Deadline** $D_i$  
  *(absolute deadline $d_i$)*
  The relative deadline is the maximum tolerated response time.

- **Tardiness**
  Measures the deadline violation.
  $0$ if $f_i \leq d_i$, otherwise $f_i - d_i$
Introduction
Task Model (continued)

- Slack time $t_s$

![Diagram showing task model with release time $r_i$, start time $s_i$, finish time $f_i$, absolute deadline $d_i$, slack time $t_s = (d_i - t - \hat{e}_i)$, and remaining execution time $\hat{e}_i$.]
Introduction
Task Model (continued)

• Preemptable Task
  A task is called **preemptable** if its execution can be suspended.
    – **Fully preemptable**: preemption can occur at any time
    – **Preemption Points**: preemption can only occur at predefined times

• Periodic Task
  A task is called **periodic**, if it is released with a fixed frequency (or period $p$).

• Aperiodic Task
  A task is called **aperiodic**, if it either has a soft deadline or no deadline at all.

• Sporadic Task
  A task is called **sporadic**, if it has a hard deadline but is released at random times.
Introduction
Feasible, Optimal Schedule & Schedulability Test

- **Feasible Schedule**

  A schedule is called *feasible*, if all tasks of the task set \( T_i, i \in \{1,2,\ldots,k\} \) that share the CPU meet their deadlines:
  \[
  O_i \leq D_i, \forall i \in \{1,2,\ldots,k\}
  \]

- **Optimal Scheduler**

  We call a scheduler *optimal* if the algorithm always produces a feasible schedule given that a feasible schedule exists for the task set.

- **Schedulability Test**

  A schedulability test verifies whether a feasible schedule exists for a particular task set.
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   c. Dynamic Scheduling (Online)

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4. Resources and Resource Access Control
Scheduling Algorithms
Overview

• **Static Scheduling (Offline)**
  A static scheduling is defined at compile time (offline). All tasks as well as important parameters (e.g. execution times) need to be known a priori.

• **Dynamic Scheduling (Online)**
  A dynamic scheduling is performed at runtime, based on the current set of active tasks and their resource dependencies.

![Diagram of Scheduling Algorithms](image-url)
Scheduling Algorithms

Overview

- **Static Priorities**
  Priority of task depends on task parameters that are known a priori (e.g. deadline or period) and does not change over runtime.

- **Dynamic Priorities**
  Priority of task changes at runtime depending on dynamic parameters (e.g. currently allocated resources).
Scheduling Algorithms
Overview

- **Preemptive**
  A scheduler is called preemptive, if it is able to interrupt the execution of a task and to re-assign the CPU.

- **Non-Preemptive**
  A scheduler is called non-preemptive if it executes a once started task until it finishes or blocks.
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Clock-Driven Scheduling

Notations and Assumptions

• The clock-driven scheduling approach is only applicable if the system is deterministic.

• Assumptions:
  – There are $n$ periodic tasks in the system.
  – The parameters of all tasks are known a priori.

• Periodic task model notation:
  – There are $n$ periodic tasks $T_i$, defined by the 4-tuple:
    $$T_i: (\phi_i; p_i; e_i; D_i)$$
    where $\phi_i$ is the phase and $p_i$ is the period of the periodic task.
  – If the phase is 0, we will omit it.
  – If the period is equal to the relative deadline, we will omit $D_i$. 
Clock-Driven Scheduling
Variable Frame Length Schedule

- A **frame** is the time interval after which the scheduler will be triggered.
- The length of a frame is called the **frame size** $f$.
- **Example of a static scheduler with a variable frame size $f$:**
  - Given are four independent periodic tasks that are executed on a single-processor system: $T_i=(p_i, e_i)$
    - $T1 = (4, 1)$
    - $T2 = (5, 1.8)$
    - $T3 = (20, 1)$
    - $T4 = (20, 2)$
Clock-Driven Scheduling
Variable Frame Length Schedule

• **Example (continued):**
  
  – *The hyperperiod* $H$ (*the least common multiple of all* $p_i$*) is 20*
  
  – *A possible static schedule is shown in the following figure (if no task is running the Idle-Task is executed):*
  
  – *The scheduler is called at times: 0, 1, 2, 3.8, 4, 6, etc.*

  ➔ *no fixed frame size*
Clock-Driven Scheduling
Fixed Frame Length Schedule

• Ideally, we want to ensure that the cyclic schedule has some desired characteristics, e.g. a constant frame size.

• An optimal, constant frame size can be computed from a task set $T_i$ by taking the following constraints into account (Baker and Shaw, 1988):
  
  – Constraint 1: The frame size should be smaller than or equal to the relative deadline $D_i$:
    \[ f \leq \min_{1 \leq i \leq k}( D_i ) \]
  
  – Constraint 2: Ideally, the frame size should be large enough to execute the longest task within one single frame:
    \[ f \geq \max_{1 \leq i \leq k}( e_i ) \]
Clock-Driven Scheduling
Fixed Frame Length Schedule

– Constraint 3: The hyperperiod $H$ should be an integer multiple of the frame size $f$:

$$F = \frac{H}{f} \text{ with } F \in \mathbb{N}$$

(The relevant frame sizes $f$ can easily be determined by computing all integer factors of the periods of the tasks)

– Constraint 4: The frame size $f$ has to be small enough to ensure that no task misses its deadline (between the release time and the deadline has to fit at least one frame):

$$2f - GCD(p_i, f) \leq D_i$$

(GCD = Greatest Common Divisor)
Constraint 4 – Explanation

\[ t + 2f \leq t_i' + D_i \]
\[ 2f - (t_i' - t) \leq D_i \]

As we are interested in the upper limit of \( f \), we have to compute the smallest possible value of \((t_i' - t)\) larger than 0: This is the greatest common divisor of \( p_i \) and \( f \):

\[ 2f - GCD(p_i, f) \leq D_i \]

**Example:**

*T with period 5*

*Frame size \( f = 3 \)*
Clock-Driven Scheduling
Fixed Frame Length Schedule

• Example:
  – Tasks \( T_i = (p_i, e_i) \): \( T_1 = (4, 1) \), \( T_2 = (5, 1.8) \), \( T_3 = (20, 1) \), \( T_4 = (20, 2) \)
    • Constraint 1: \( f \leq 4 \)
    • Constraint 2: \( f \geq 2 \)
    • Constraint 3: \( f = \{2, 4, 5, 10, 20\} \rightarrow \{5, 10, 20\} \text{ can be ignored due to constraint 1} \)
    • Constraint 4:
      – \( f = 2 \):
        » \( T_1 \): 4 - GCD(4,2) = 2 \leq 4 \text{ (ok)}
        » \( T_2 \): 4 - GCD(5,2) = 3 \leq 5 \text{ (ok)}
        » \( T_3 \): 4 - GCD(20,2) = 2 \leq 20 \text{ (ok)}
        » \( T_4 \): 4 - GCD(20,2) = 2 \leq 20 \text{ (ok)}
      – \( f = 4 \):
        » \( T_1 \): 8 - GCD(4,4) = 4 \leq 4 \text{ (ok)}
        » \( T_2 \): 8 - GCD(5,4) = 7 \leq 5 \text{ (not ok)}

\( \rightarrow \text{Only feasible frame size: } f = 2 \)
Clock-Driven Scheduling
Fixed Frame Length Schedule

• Example (continued):
  – Tasks \( T_i = (p_i, e_i) \): \( T_1 = (4, 1), T_2 = (5, 1.8), T_3 = (20, 1), T_4 = (20, 2) \)
Clock-Driven Scheduling
Fixed Frame Length Schedule

• Sometimes the given task set cannot meet the four frame size constraints simultaneously.

• Example:
  Consider the task set: \( T_i = (p_i, e_i, D_i) \)
  \( T_1 = (4, 1), \ T_2 = (5, 2, 7), \ T_3 = (20, 5) \)
  – To satisfy constraint 1: \( f \leq 4 \)
  – To satisfy constraint 2: \( f \geq 5 \)
  \( \rightarrow \) This is not possible!!!

• Solution: Partition a task into subtasks.
Clock-Driven Scheduling
Fixed Frame Length Schedule

- E.g. partitioning $T_3 = (20, 5)$ in:
  - $T_{3,1} = (20, 1)$,
  - $T_{3,2} = (20, 3)$ and
  - $T_{3,3} = (20, 1)$

yields a frame size of 4.
Clock-Driven Scheduling
Fixed Frame Length Schedule, Aperiodic Tasks

• Aperiodic tasks are scheduled after all tasks with hard deadline requirements are scheduled.

• To improve the response time of aperiodic tasks, they should be executed before the periodic tasks.

→ This is called slack-stealing
Clock-Driven Scheduling
Fixed Frame Length Schedule, Aperiodic Tasks

- Slack-Stealing Example

\[ A_3 \]
\( (e_3 = 2) \)

\[ A_1 \]
\( (e_1 = 1.5) \)

\[ A_2 \]
\( (e_2 = 0.5) \)

Without aperiodic jobs

Aperiodic Jobs
no slack-stealing

Aperiodic Jobs
Slack-stealing

Average Response Time of A1, A2 and A3: 4.5

Average Response Time of A1, A2 and A3: 2.5
Clock-Driven Scheduling
Fixed Frame Length Schedule, Sporadic Tasks

• Sporadic tasks have, similar to periodic tasks, hard deadlines.

• If more than one sporadic task is waiting, they should be ordered on the Earliest-Deadline-First (EDF) basis.

• Whether a sporadic task $S(d, e)$ is accepted or rejected by the scheduler is determined by an **acceptance** test.

  – **Acceptance Test:**
    The sporadic task $S$ is accepted if the accumulated slack times from frame $t$ to $l$
    $\sigma_c(t, l)$ is greater than or equal to the execution time of the sporadic task $S(d,e)$.

    $$e \leq \sigma_c(t, l)$$

    $\sigma_c(t, l) = \sigma_t + \sigma_{t+1} + \ldots + \sigma_{l-1} + \sigma_l$
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Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate Monotonic Algorithm

• In the rate monotonic (RM) algorithm, task priorities depend on the task rate \((1/p_i)\)
  \(\rightarrow\) the higher the rate, the higher the priority.

• Example:
  - Task-Set: \(T_i = (p_i, e_i)\)
    - \(T_1=(4,1) \rightarrow \text{Priority high}\)
    - \(T_2=(5,2) \rightarrow \text{Priority medium}\)
    - \(T_3=(20,5) \rightarrow \text{Priority low}\)
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate Monotonic Algorithm

• Example: $T_1=(4,1)$, $T_2=(5,2)$, $T_3=(20,5)$
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Deadline Monotonic Algorithm

• In the **deadline monotonic** (DM) algorithm, task priorities depend on the *relative* task deadline \( D_i \)
  \( \rightarrow \) the shorter the relative deadline, the higher the priority.

• **Example:**
  
  - **Task-Set:** \( T_i = (\phi_i, p_i, e_i, D_i) \)
    
    - \( T_1 = (50, 50, 25, 100) \rightarrow Priority low \)
    - \( T_2 = (0, 62.5, 10, 20) \rightarrow Priority high \)
    - \( T_3 = (0, 125, 25, 50) \rightarrow Priority medium \)
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Deadline Monotonic Algorithm

• Example (continued): $T_i = (\phi_i, p_i, e_i, D_i)$
  $T_1 = (50, 50, 25, 100)$, $T_2 = (0, 62.5, 10, 20)$, $T_3 = (0, 125, 25, 50)$
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate vs. Deadline Monotonic

• Important notes:
  – If the relative deadlines and the periods of all tasks are proportional, the rate and deadline monotonic algorithms are identical.
  – When the relative deadlines are arbitrary, the DM algorithm can sometimes produce a feasible schedule when the RM algorithm fails.
  – The RM algorithm always fails when the DM algorithm fails.
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate vs. Deadline Monotonic

- Previous DM example, scheduled by a RM scheduler:
  - DM resulted in feasible schedule, RM fails.
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

• The Earliest-Deadline-First (EDF) algorithm assigns priorities to tasks according to their absolute deadlines $d_i$.

→ The earlier the deadline, the higher the priority.

• Example:

  – Given task set: $T_i=(p_i, e_i)$
    
      • $T_1 = (2, 0.9)$
      • $T_2 = (5, 2.3)$
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- *Example (continued):* \( T_1 = (2, 0.9), T_2 = (5, 2.3) \)

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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**Priority-Driven Scheduling**

*Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm*

- **Example (continued):** \( T_1 = (2, 0.9), \ T_2 = (5, 2.3) \)

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Diagram showing the scheduling of tasks with dynamic priorities.
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

• Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

• The Least-Slack-Time-First algorithm assigns priorities to tasks according to their slack time.
  → the smaller the slack time, the higher the priority

• Definition of slack time (recapitulation):

Note:
– Slack time of currently running processes is constant.
– Slack time of waiting processes shortens.
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- **Example** ($T_i = (p_i, e_i)$): $T_1 = (2, 0.8), T_2 = (5, 1.5), T_3 = (5.1, 1.5)$

- **Slack-Time**: $t_s = d - t - \hat{e}$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- Example \( (T_i=(p_i, e_i)) \): \( T_1 = (2, 0.8) \), \( T_2 = (5, 1.5) \), \( T_3 = (5.1, 1.5) \)

- Slack-Time: \( t_s = d - t - \hat{e} \)

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\( \hat{e} = 0.8 \)
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- **Example** \((T_i=(p_i, e_i))\): \(T_1 = (2, 0.8), T_2 = (5, 1.5), T_3 = (5.1, 1.5)\)

- **Slack-Time**: \(t_s = d - t - \hat{e}\)

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- Example \(T_i=(p_i, e_i)\): \(T_1 = (2, 0.8), T_2 = (5, 1.5), T_3 = (5.1, 1.5)\)

- Slack-Time: \(t_s = d - t - \hat{e}\)

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\[T_1 (e_1 = 0.8)\]
\[T_2 (e_2 = 1.5)\]
\[T_3 (e_3 = 1.5)\]
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- **Example** \( (T_i=(p_i, e_i))\): \( T_1 = (2, 0.8),\ T_2 = (5, 1.5),\ T_3 = (5.1, 1.5) \)

- **Slack-Time**: \( t_s = d - t - \hat{e} \)

| \( t \) | \( d \) / \( \hat{e} \) / \( t_s \) |
|---|---|---|
| \( T_1 \) | \( T_2 \) | \( T_3 \) |
| 0 | 2 / 0.8 / 1.2 | 5 / 1.5 / 3.5 | 5.1 / 1.5 / 3.6 |
| 0.8 | - | 5 / 1.5 / 2.7 | 5.1 / 1.5 / 2.8 |
| 2 | 4 / 0.8 / 1.2 | 5 / 0.3 / 2.7 | 5.1 / 1.5 / 1.6 |
| 2.8 | - | 5 / 0.3 / 1.9 | 5.1 / 1.5 / 0.8 |
| 4 | 6 / 0.8 / 1.2 | 5 / 0.3 / 0.7 | 5.1 / 0.3 / 0.8 |
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Summary EDF and LST

• Both, EDF and LST are optimal if:
  – Preemption of tasks is allowed
  – Tasks do not contend for resources
  – A single processor system is used

• EDF does not require knowledge of execution times, LST does → huge drawback
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Summary EDF and LST

• Both, EDF and LST are optimal if:
  – Preemption of tasks is allowed
  – Tasks do not contend for resources
  – A single processor system is used

• Proof (EDF):
  – Idea: Each valid schedule can be transformed in EDF
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**Schedulability Testing**

**Introduction**

- A test to validate that a given set of tasks can meet its hard deadlines when scheduled according to a specific scheduling algorithm is called *schedulability* test.
Schedulability Testing
DM and RM Algorithms

• A task set of \( n \) tasks can be feasibly scheduled on one processor by the RM algorithm if the following utilization condition holds (Liu und Layland 1973):

\[
U = \sum_{i=1}^{n} \frac{e_i}{p_i} \leq n(2^{1/n} - 1)
\]

• Note: The tasks have to be:
  – independent,
  – preemptable, and
  – periodic.

Recapitulation: If the relative deadlines of all task in a given task set are proportional to the periods, the DM algorithm is identical to the RM algorithm and the above condition can also be used to perform a schedulability test for the DM algorithm.
Schedulability Testing
DM and RM Algorithms

• Example:

<table>
<thead>
<tr>
<th>Task</th>
<th>$p_i$</th>
<th>$e_i$</th>
<th>$u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>1.75</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>0.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Sum: 0.62**

*Total utilization $U = 0.62 \leq 0.743 \rightarrow$ task set can be feasibly scheduled by the RM algorithm.*
Schedulability Testing
DM and RM Algorithms

- **Important:**
  The presented condition is not a necessary condition !!!
  → Even if the utilization of a task set exceeds the condition, a feasible RM schedule might exist.
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

• For a sorted task set $T_i$ (i.e. $T_0 =$ task with highest priority, $T_i =$ task with lowest priority), we can perform a time-demand analysis, by (Lehoczky et al., 1989)
  1. computing the time-demand of all tasks $T_i$, according to:

$$w_i(t) = e_i + \sum_{k=1}^{i-1} \left[ \frac{t}{p_k} \right] e_k \text{ for } 0 < t \leq p_i$$

  2. checking whether the inequality

$$w_i(t) \leq t$$

is satisfied for values of $t$ that are equal to

$$t = j p_k ; k = 1, 2, \ldots, i ; j = 1, 2, \ldots, \left\lfloor \min(p_i, D_i) / p_k \right\rfloor$$

If this inequality is satisfied at one of these instants, $T_i$ is schedulable.
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

- **Example:**
  \( T_1=(\phi_1, 3, 1); T_2=(\phi_2, 5, 1.5), T_3=(\phi_3, 7, 1.25), T_4=(\phi_4, 9, 0.5) \)
  
  - \( w_1: \)
    - \( w_1(3) = 1 \leq 3 \rightarrow OK \)
  
  - \( w_2: \)
    - \( w_2(3) = 1.5 + 1 = 2.5 \leq 3 \rightarrow OK \)
  
  - \( w_3: \)
    - \( w_3(3) = 1.25 + 1 + 1.5 = 3.75 > 3 \rightarrow Not OK \)
    - \( w_3(5) = 1.25 + 2 + 1.5 = 4.75 \leq 5 \rightarrow OK \)
  
  - \( w_4: \)
    - \( w_4(3) = 0.5 + 1 + 1.5 + 1.25 = 4.25 > 3 \rightarrow Not OK \)
    - \( w_4(5) = 0.5 + 2 + 1.5 + 1.25 = 5.25 > 5 \rightarrow Not OK \)
    - \( w_4(6) = 0.5 + 2 + 3 + 1.25 = 6.75 > 6 \rightarrow Not OK \)
    - \( w_4(7) = 0.5 + 3 + 3 + 1.25 = 7.75 > 7 \rightarrow Not OK \)
    - \( w_4(9) = 0.5 + 3 + 3 + 2.5 = 9 \leq 9 \rightarrow OK \)
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

- Example (continued):

**Graphical demonstration of time-demand analysis**

![Graphical representation of time-demand analysis](image)
Schedulability Testing

EDF Algorithm

- Task density:
  \[ \text{density}_k = \frac{e_k}{\min(D_k, p_k)} \]

- A set of
  - independent,
  - periodic, and
  - preemptable

tasks can be *feasibly* scheduled by the EDF algorithm on one processor if the task set density is less or equal to 1:

\[
\sum_{k=1}^{n} \frac{e_k}{\min(D_k, p_k)} \leq 1
\]

Note: This is only a sufficient condition. Even if inequality is not satisfied, a feasible schedule might exist.
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Resources and Resource Access Control

Introduction

• If resources can only be used in a mutual exclusive manner, resource contentions occur that can lead to system failures.

• Effects of resource contentions:
  – Priority Inversions
  – Deadlocks
Resources and Resource Access Control

Effects of Resource Contention: Priority Inversion

- The phenomenon that a lower-priority task blocks a higher-priority task is called **priority inversion**.

![Diagram showing priority inversion](image-url)
Resources and Resource Access Control
Effects of Resource Contention: Uncontrolled Priority Inversion

- Uncontrolled (or Unbounded) Priority Inversion
  A medium priority task can block a high priority task forever.

Uncontrolled priority inversion can only occur if the task set contains more than 2 tasks.
Resources and Resource Access Control

Effects of Resource Contention: Deadlock

- Consider two tasks $T_1$ and $T_2$ and two resources $R_1$ and $R_2$.
  - $T_1$ holds $R_1$, requests $R_2$
  - $T_2$ holds $R_2$, requests $R_1$
  \[\rightarrow \] Deadlock
Resources and Resource Access Control

Nonpreemptive Critical Section (NPCS) Protocol

• Simple way to control access to a resource is to schedule all critical sections nonpreemptively:

If a task request a resource, it is always allocated the resource and executes with the highest priority.

→ This protocol is called the Nonpreemptive Critical Section (NPCS) protocol

• As no preemption takes place, no deadlock or priority inversion can occur!!!

• Shortcoming: Every task can be blocked by every lower-priority task, even if there is no resource conflict.
Resources and Resource Access Control

Basic Priority Inheritance Protocol (BPIP)

• The basic priority inheritance protocol (BPIP) prevents uncontrolled priority inversions but not deadlocks.

→ This is achieved by raising the current priority $\pi_l(t)$ of a lower-priority task to a higher (inherited) priority $\pi_h(t)$ of another task.

• BPIP rules:
  – *Scheduling Rule*: Ready tasks are scheduled preemptively in a priority-driven manner according to their current priorities. At the release time, the current priority $\pi(t)$ is equal to the assigned priority (the priority determined by the scheduling algorithm).
Resources and Resource Access Control

Basic Priority Inheritance Protocol (BPIP)

• BPIP rules (continued):

  – Allocation Rule: When a task \( T \) requests a resource \( R \) at time \( t \),
    a) if \( R \) is free, \( R \) is allocated to \( T \) until \( T \) releases the resource, and
    b) if \( R \) is not free, the request is denied and \( T \) is blocked.

  – Priority-Inheritance Rule: When the requesting task \( T \) becomes blocked, the task \( T_i \) which blocks \( T \) inherits the current priority of \( T \) until it releases the resource. At that time, the priority of \( T_i \) returns to the value it had at the time when it acquired \( R \).
Resources and Resource Access Control
Basic Priority Inheritance Protocol (BPIP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T5 executes with priority 5</td>
</tr>
<tr>
<td>1</td>
<td>T5 is granted resource “black”</td>
</tr>
<tr>
<td>2</td>
<td>T4 released, preempts T5</td>
</tr>
<tr>
<td>3</td>
<td>T4 is granted resource “dotted”</td>
</tr>
<tr>
<td>4</td>
<td>T3 released, preempts T4</td>
</tr>
<tr>
<td>5</td>
<td>T2 released, preempts T3</td>
</tr>
<tr>
<td>6</td>
<td>T2 requests resource “black”, T5 inherits priority of T2 and executes</td>
</tr>
<tr>
<td>7</td>
<td>T1 released, preempts T5</td>
</tr>
</tbody>
</table>
# Resources and Resource Access Control

## Basic Priority Inheritance Protocol (BPIP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>T1 requests resource “dotted”, T4 inherits priority of T1</td>
</tr>
<tr>
<td>9</td>
<td>T4 requests resource “black”, T5 inherits priority and executes</td>
</tr>
<tr>
<td>11</td>
<td>T5 releases resource “black”, T4 continues</td>
</tr>
<tr>
<td>13</td>
<td>T4 releases resource “dotted”, T1 acquires resource “dotted” and continues</td>
</tr>
<tr>
<td>15</td>
<td>T1 completes, T2 is granted resource “black” and executes</td>
</tr>
<tr>
<td>17</td>
<td>T2 completes, afterwards T3, T4 and T5 execute and complete</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
Resources and Resource Access Control

Basic Priority Ceiling Protocol (BPCP)

• The basic priority ceiling protocol (BPCP) extends the BPIP to prevent deadlocks and to further reduce the blocking time.

• **Priority Ceiling**: The priority ceiling $\Pi(R_i)$ of a resource $R_i$ is the highest priority of all the tasks that require $R_i$.
  
  – *Example (based on previous slide):* $\Pi(B) = 2$, $\Pi(D) = 1$

• **Current Priority Ceiling (or simply ceiling)**: The ceiling $\hat{\Pi}(t)$ is equal to the highest priority ceiling of the resources currently in use. If all resources are free, the ceiling is equal to $\Omega$, a non-existing priority lower than any other priority.

  – Example (based on previous slide):
    
    - In $(1,3]$, resource „black“ is used; hence the ceiling is 2
    - In $(3,13]$, resource „dotted“ is used; hence the ceiling is 1
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

• BPCP rules:
  – *Scheduling Rule*:
    a) At its release time, the current task priority $\pi(t)$ is equal to its assigned priority.
    b) Every ready task is scheduled preemptively and in a priority-driven manner, depending on its current priority $\pi(t)$.
  – *Allocation rule*:
    Whenever a task $T$ requests a resource $R$ at time $t$, one of the following conditions occurs:
    a) $R$ is held by another task $\rightarrow T$ blocks
    b) $R$ is free
      a) If the priority $\pi(t)$ of $T$ is higher than the current priority ceiling, $R$ is allocated to $T$.
      b) If the priority of $T$ is *not* higher than the ceiling, $R$ is allocated to $T$ only if $T$ is holding the resource whose priority ceiling is equal to the ceiling; otherwise $T$ blocks.
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

• BPCP rules:

  – *Priority Inheritance Rule*: When $T$ becomes blocked, the task $T_i$ that blocks $T$ inherits the current priority of $T$. $T_i$ executes at its inherited priority until the time when it releases every resource whose priority ceiling is equal to or higher than the priority of $T$; at that time, the priority of $T_i$ returns to the value it had when it was granted the resource.