Real-Time Systems

Part 7: Scheduling
Content

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2. Scheduling Algorithms
   a. Overview
   b. Offline Schedulers
   c. Online Schedulers

3. Schedulability Testing

4. Resources and Resource Access Control
Literature

- Jane W. S. Liu, Real-Time Systems, 2000
- Fridolin Hofmann: Betriebssysteme - Grundkonzepte und Modellvorstellungen, 1991
- Klaus Gresser, Echtzeitnachweis ereignisgesteueter Realzeitsysteme, Dissertation, TUM, 1993

Journals:

- Giorgio C. Buttazzo: Rate Monotonic vs. EDF: Judgement Day (http://www.cas.mcmaster.ca/~downd/rtsj05-rmedf.pdf)
Introduction
Scheduler and Dispatcher

- **Scheduler:**
  
  If a resource is to be used by many consumers, access to the resource has to be coordinated. This resource *allocation* is performed by a **scheduler**.

  In computer systems, the term scheduler often refers to the CPU scheduler which controls the allocation of the CPU to *tasks*.

- **Dispatcher:**
  
  While the scheduler plans the CPU allocation, the dispatcher executes the scheduler plan by:
  
  - Switching the context
  - Switching to user mode
  - Jumping to the proper location in the user program to restart it
Introduction

Task Model

We introduce the following model for a task:

- **Release Time (or arrival time) $r_i$**
  Earliest time at which task $i$ is enabled.

- **Start Time $s_i$**
  Time at which execution of task starts.

- **Finish Time $f_i$**
  Time at which task completes execution.

- **Response Time $O_i$**
  Interval between release and finish time.
Introduction
Task Model (continued)

We introduce the following model for a task:

- **Execution Time** $e_i$  
  *(remaining execution time $\hat{e}_i$ – see next slide)*
  Total time of task execution (does not include durations where the task was blocked).

- **Relative Deadline** $D_i$  
  *(absolute deadline $d_i$)*
  The relative deadline is the maximum tolerated response time.

- **Tardiness**
  Measures the deadline violation.
  $0$ if $f_i \leq d_i$, otherwise $f_i - d_i$
Introduction
Task Model (continued)

• Slack time $t_s$
Introduction
Task Model (continued)

• Preemptable Task
  A task is called **preemptable** if its execution can be suspended.
    - **Fully preemptable**: preemption can occur at any time
    - **Preemption Points**: preemption can only occur at predefined times

• Periodic Task
  A task is called **periodic**, if it is released with a fixed frequency (or period $p$).

• Aperiodic Task
  A task is called **aperiodic**, if it either has a soft deadline or no deadline at all.

• Sporadic Task
  A task is called **sporadic**, if it has a hard deadline but is released at random times.
Introduction
Feasible, Optimal Schedule & Schedulability Test

• Feasible Schedule

A schedule is called **feasible**, if all tasks of the task set $T_i, i \in \{1, 2, \ldots, k\}$ that share the CPU meet their deadlines:

$$O_i \leq D_i, \forall i \in \{1, 2, \ldots, k\}$$

• Optimal Scheduler

We call a scheduler **optimal** if the algorithm always produces a feasible schedule given that a feasible schedule exists for the task set.

• Schedulability Test

A schedulability test varifies whether a feasible schedule exists for a particular task set.
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Scheduling Algorithms

Overview

• Static Scheduling (Offline)
  A static scheduling is defined at compile time (offline). All tasks as well as important parameters (e.g. execution times) need to be known a priori.

• Dynamic Scheduling (Online)
  A dynamic scheduling is performed at runtime, based on the current set of active tasks and their resource dependencies.
Scheduling Algorithms

Overview

- **Static Priorities**
  Priority of task depends on task parameters that are known a priori (e.g. deadline or period) and does not change over runtime.

- **Dynamic Priorities**
  Priority of task changes at runtime depending on dynamic parameters (e.g. currently allocated resources).
Scheduling Algorithms

Overview

- **Preemptive**
  A scheduler is called preemptive, if it is able to interrupt the execution of a task and to re-assign the CPU.

- **Non-Preemptive**
  A scheduler is called non-preemptive if it executes a once started task until it finishes or blocks.

![Diagram of Scheduling Algorithms]

- Static Scheduling (Offline)
  - Preemptive
  - Non-Preemptive

- Dynamic Scheduling (Online)
  - Static Priorities
    - Preemptive
    - Non-Preemptive
  - Dynamic Priorities
    - Preemptive
    - Non-Preemptive
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Clock-Driven Scheduling

Notations and Assumptions

- The clock-driven scheduling approach is only applicable if the system is deterministic.

- **Assumptions:**
  - There are $n$ periodic tasks in the system.
  - The parameters of all tasks are known a priori.

- **Periodic task model notation:**
  - There are $n$ periodic tasks $T_i$, defined by the 4-tuple:
    $$ T_i: (\phi_i; p_i; e_i; D_i) $$
    where $\phi_i$ is the phase and $p_i$ is the period of the periodic task.
  - If the phase is 0, we will omit it.
  - If the period is equal to the relative deadline, we will omit $D_i$. 
Clock-Driven Scheduling
Variable Frame Length Schedule

• A \textit{frame} is the time interval after which the scheduler will be triggered.

• The length of a frame is called the \textit{frame size} $f$.

• \textit{Example of a static scheduler with a \textit{variable} frame size $f$:}
  
  – \textit{Given are four independent periodic tasks that are executed on a single-processor system: $T_i=(p_i, e_i)$}
    
    • $T_1 = (4, 1)$
    • $T_2 = (5, 1.8)$
    • $T_3 = (20, 1)$
    • $T_4 = (20, 2)$
Clock-Driven Scheduling
Variable Frame Length Schedule

• Example (continued):
  – The hyperperiod $H$ (the least common multiple of all $p_i$) is 20
  – A possible static schedule is shown in the following figure (if no task is running the Idle-Task is executed):
  – The scheduler is called at times: 0, 1, 2, 3.8, 4, 6, etc.
  \[\rightarrow\text{no fixed frame size}\]
Clock-Driven Scheduling
Fixed Frame Length Schedule

• Ideally, we want to ensure that the cyclic schedule has some desired characteristics, e.g. a constant frame size.

• An optimal, constant frame size can be computed from a task set $T_i$ by taking the following constraints into account (Baker and Shaw, 1988):
  
  – Constraint 1: The frame size should be smaller than or equal to the relative deadline $D_i$:
    
    $$ f \leq \min_{1 \leq i \leq k}(D_i) $$
  
  – Constraint 2: Ideally, the frame size should be large enough to execute the longest task within one single frame:
    
    $$ f \geq \max_{1 \leq i \leq k}(e_i) $$
Clock-Driven Scheduling

Fixed Frame Length Schedule

– Constraint 3: The hyperperiod $H$ should be an integer multiple of the frame size $f$:

$$ F = \frac{H}{f} \text{ with } F \in N $$

(The relevant frame sizes $f$ can easily be determined by computing all integer factors of the periods of the tasks)

– Constraint 4: The frame size $f$ has to be small enough to ensure that no task misses its deadline (between the release time and the deadline has to fit at least one frame):

$$ 2f - GCD(p_i, f) \leq D_i $$

(GCD = Greatest Common Divisor)
Clock-Driven Scheduling
Fixed Frame Length Schedule

– Constraint 4 – Explanation

\[ t + 2f \leq t'_i + D_i \]
\[ 2f - (t'_i - t) \leq D_i \]

As we are interested in the upper limit of \( f \), we have to compute the smallest possible value of \( (t'_i - t) \) larger than 0: This is the greatest common divisor of \( p_i \) and \( f \): \[ 2f - \text{GCD}(p_i, f) \leq D_i \]

**Example:**

*\( T \) with period 5

Frame size \( f = 3 \)
Clock-Driven Scheduling
Fixed Frame Length Schedule

- Example:

  Tasks \((T_i = (p_i, e_i))\): \(T_1=(4,1), T_2=(5,1.8), T_3=(20,1), T_4=(20,2)\)

  - Constraint 1: \(f \leq 4\)
  - Constraint 2: \(f \geq 2\)
  - Constraint 3: \(f = \{2,4,5,10,20\} \Rightarrow \{5,10,20\} \text{ can be ignored due to constraint 1}\)
  - Constraint 4:
    - \(f = 2:\)
      - \(T_1: 4 - \text{GCD}(4,2) = 2 \leq 4 \text{ (ok)}\)
      - \(T_2: 4 - \text{GCD}(5,2) = 3 \leq 5 \text{ (ok)}\)
      - \(T_3: 4 - \text{GCD}(20,2) = 2 \leq 20 \text{ (ok)}\)
      - \(T_4: 4 - \text{GCD}(20,2) = 2 \leq 20 \text{ (ok)}\)
    - \(f = 4:\)
      - \(T_1: 8 - \text{GCD}(4,4) = 4 \leq 4 \text{ (ok)}\)
      - \(T_2: 8 - \text{GCD}(5,4) = 7 \leq 5 \text{ (not ok)}\)

\(\Rightarrow\) Only feasible frame size: \(f = 2\)
Clock-Driven Scheduling

Fixed Frame Length Schedule

- Example (continued):
  - Tasks \( T_i=(p_i, e_i) \): \( T_1=(4,1) \), \( T_2=(5, 1.8) \), \( T_3=(20,1) \), \( T_4=(20,2) \)
Clock-Driven Scheduling
Fixed Frame Length Schedule

- Sometimes the given task set cannot meet the four frame size constraints simultaneously.

- Example:
  Consider the task set: \( T_i = (p_i, e_i, D_i) \)
  \( T_1 = (4, 1), \ T_2 = (5, 2, 7), \ T_3 = (20, 5) \)
  - To satisfy constraint 1: \( f \leq 4 \)
  - To satisfy constraint 2: \( f \geq 5 \)
  \( \Rightarrow \) This is not possible!!!

- Solution: Partition a task into subtasks.
Clock-Driven Scheduling
Fixed Frame Length Schedule

- E.g. partitioning $T_3 = (20, 5)$ in:
  - $T_{3,1} = (20, 1)$
  - $T_{3,2} = (20, 3)$ and
  - $T_{3,3} = (20, 1)$

yields a frame size of 4.
Clock-Driven Scheduling
Fixed Frame Length Schedule, Aperiodic Tasks

• Aperiodic tasks are scheduled after all tasks with hard deadline requirements are scheduled.

• To improve the response time of aperiodic tasks, they should be executed before the periodic tasks.

  → This is called slack-stealing
Clock-Driven Scheduling
Fixed Frame Length Schedule, Aperiodic Tasks

- Slack-Stealing Example

\[ A_3 \quad (e_3 = 2) \]
\[ A_2 \quad (e_2 = 0.5) \]
\[ A_1 \quad (e_1 = 1.5) \]

Without aperiodic jobs

Aperiodic Jobs **no** slack-stealing

Aperiodic Jobs **Slack-stealing**

Average Response Time of A1, A2 and A3: 4.5

Average Response Time of A1, A2 and A3: 2.5
Clock-Driven Scheduling
Fixed Frame Length Schedule, Sporadic Tasks

• Sporadic tasks have, similar to periodic tasks, hard deadlines.

• If more than one sporadic task is waiting, they should be ordered on the Earliest-Deadline-First (EDF) basis.

• Whether a sporadic task $S(d, e)$ is accepted or rejected by the scheduler is determined by an acceptance test.

  - **Acceptance Test:**
    The sporadic task $S$ is accepted if the accumulated slack times from frame $t$ to $l$
    $\sigma_c(t, l)$ is greater than or equal to the execution time of the sporadic task $S(d,e)$.

    $$e \leq \sigma_c(t, l)$$
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Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate Monotonic Algorithm

• In the **rate monotonic** (RM) algorithm, task priorities depend on the task rate \(1/p_i\)
  → the higher the rate, the higher the priority.

• **Example:**
  
  – Task-Set: \(T_i = (p_i, e_i)\)
    
    • \(T_1 = (4, 1) \rightarrow \text{Priority high}\)
    • \(T_2 = (5, 2) \rightarrow \text{Priority medium}\)
    • \(T_3 = (20, 5) \rightarrow \text{Priority low}\)
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate Monotonic Algorithm

- Example: $T_1=(4,1)$, $T_2=(5,2)$, $T_3=(20,5)$
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Deadline Monotonic Algorithm

• In the deadline monotonic (DM) algorithm, task priorities depend on the relative task deadline $D_i$ → the shorter the relative deadline, the higher the priority.

• Example:
  
  – Task-Set: $T_i = (\phi_i, p_i, e_i, D_i)$
    
    • $T_1=(50, 50, 25, 100)$ → Priority low
    • $T_2=(0, 62.5, 10, 20)$ → Priority high
    • $T_3=(0, 125, 25, 50)$ → Priority medium
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Deadline Monotonic Algorithm

- Example (continued): \( T_i = (\phi_i, p_i, e_i, D_i) \)
  \( T_1 = (50, 50, 25, 100), T_2 = (0, 62.5, 10, 20), T_3 = (0, 125, 25, 50) \)
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate vs. Deadline Monotonic

• Important notes:
  – If the relative deadlines and the periods of all tasks are proportional, the rate and deadline monotonic algorithms are identical.
  – When the relative deadlines are arbitrary, the DM algorithm can sometimes produce a feasible schedule when the RM algorithm fails.
  – The RM algorithm always fails when the DM algorithm fails.
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate vs. Deadline Monotonic

- Previous DM example, scheduled by a RM scheduler:
  - DM resulted in feasible schedule, RM fails.

![Diagram showing priority-driven scheduling with periodic tasks and static priorities.](image-url)
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

• The Earliest-Deadline-First (EDF) algorithm assigns priorities to tasks according to their **absolute** deadlines $d_i$.
  → The earlier the deadline, the higher the priority.

• **Example:**
  
  – Given task set: $T_i=(p_i, e_i)$

    • $T_1 = (2, 0.9)$
    • $T_2 = (5, 2.3)$
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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$t_0$ $T_1$ $T_2$

$T_1 = T_{1,1}$

$T_2$

0 2 4 6 8
Priority-Driven Scheduling

Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): \( T_1 = (2, 0.9), T_2 = (5, 2.3) \)

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\( r_1 \)

\( T_1 \)

\( T_{1,1} \)

\( T_2 \)

\( T_{2,1} \)

0 2 4 6 8

\( t \)
Priority-Driven Scheduling

Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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**Priority-Driven Scheduling**

*Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm*

- **Example (continued):** $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
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• Example (continued): $T_1 = (2, 0.9)$, $T_2 = (5, 2.3)$
Priority-Driven Scheduling
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Priority-Driven Scheduling

Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9)$, $T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

• The Least-Slack-Time-First algorithm assigns priorities to tasks according to their slack time.
→ the smaller the slack time, the higher the priority

• Definition of slack time (recapitulation):

Note:
– Slack time of currently running processes is constant.
– Slack time of waiting processes shortens.
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

• Example \((T_i=(p_i, e_i))\): \(T_1 = (2, 0.8), T_2 = (5, 1.5), T_3 = (5.1, 1.5)\)

• Slack-Time: \(t_s = d - t - \hat{e}\)

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Priority-Driven Scheduling

Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

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\[d = t - \hat{e}\]
Priority-Driven Scheduling

Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

• Example ($T_i = (p_i, e_i)$): $T_1 = (2, 0.8)$, $T_2 = (5, 1.5)$, $T_3 = (5.1, 1.5)$

• Slack-Time: $t_s = d - t - \hat{e}$

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<td></td>
<td>$T_1$</td>
</tr>
<tr>
<td>0</td>
<td>2 / 0.8 / 1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>4 / 0.8 / 1.2</td>
</tr>
<tr>
<td>2.8</td>
<td>-</td>
</tr>
</tbody>
</table>
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- **Example** \( T_i=(p_i, e_i) \): \( T_1 = (2, 0.8) \), \( T_2 = (5, 1.5) \), \( T_3 = (5.1, 1.5) \)

- **Slack-Time**: \( t_s = d - t - \hat{e} \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( d / \hat{e} / t_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T_1 )</td>
</tr>
<tr>
<td>0</td>
<td>2 / 0.8 / 1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>4 / 0.8 / 1.2</td>
</tr>
<tr>
<td>2.8</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>6 / 0.8 / 1.2</td>
</tr>
</tbody>
</table>
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Summary EDF and LST

• Both, EDF and LST are optimal if:
  – Preemption of tasks is allowed
  – Tasks do not contend for resources
  – A single processor system is used

• EDF does not require knowledge of execution times, LST does
  → huge drawback
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Schedulability Testing

Introduction

• A test to validate that a given set of tasks can meet its hard deadlines when scheduled according to a specific scheduling algorithm is called *schedulability* test.
A task set of \( n \) tasks can be feasibly scheduled on one processor by the RM algorithm if the following utilization condition holds (Liu und Layland 1973):

\[
U = \sum_{i=1}^{n} \frac{e_i}{p_i} \leq n(2^{1/n} - 1)
\]

Note: The tasks have to be:

- independent,
- preemptable, and
- periodic.

Recapitulation: If the relative deadlines of all task in a given task set are proportional to the periods, the DM algorithm is identical to the RM algorithm and the above condition can also be used to perform a schedulability test for the DM algorithm.
Schedulability Testing
DM and RM Algorithms

• Example:

<table>
<thead>
<tr>
<th>Task</th>
<th>$p_i$</th>
<th>$e_i$</th>
<th>$u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>1.75</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>0.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Sum: 0.62**

Total utilization $U=0.62 \leq 0.743 \rightarrow$ task set can be feasibly scheduled by the RM algorithm.
Schedulability Testing
DM and RM Algorithms

• Important:
The presented condition is not a necessary condition !!!
⇒ Even if the utilization of a task set exceeds the condition, a feasible RM schedule might exist.

• A schedulability test of such a task set, scheduled by a fixed-priority algorithm, can be performed by the time-demand analysis.
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

• For a sorted task set $T_i$ (i.e. $T_0 =$ task with highest priority, $T_i =$ task with lowest priority), we can perform a time-demand analysis, by (Lehoczky et al., 1989)

1. computing the time-demand of all tasks $T_i$, according to:

$$w_i(t) = e_i + \sum_{k=1}^{i-1} \left[ \frac{t}{p_k} \right] e_k \quad \text{for } 0 < t \leq p_i$$

2. checking whether the inequality

$$w_i(t) \leq t$$

is satisfied for values of $t$ that are equal to

$$t = j p_k; k = 1, 2, \ldots, i; j = 1, 2, \ldots, \left\lfloor \min(p_i, D_i) / p_k \right\rfloor$$

If this inequality is satisfied at one of these instants, $T_i$ is schedulable.
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

- **Example:**
  \( T_1 = (\phi_1, 3, 1); T_2 = (\phi_2, 5, 1.5), T_3 = (\phi_3, 7, 1.25), T_4 = (\phi_4, 9, 0.5) \)
  
  - \( w_1 \):
    - \( w_1(3) = 1 \leq 3 \rightarrow OK \)
  
  - \( w_2 \):
    - \( w_2(3) = 1.5 + 1 = 2.5 \leq 3 \rightarrow OK \)
  
  - \( w_3 \):
    - \( w_3(3) = 1.25 + 1 + 1.5 = 3.75 > 3 \rightarrow Not OK \)
    - \( w_3(5) = 1.25 + 2 + 1.5 = 4.75 \leq 5 \rightarrow OK \)
  
  - \( w_4 \):
    - \( w_4(3) = 0.5 + 1 + 1.5 + 1.25 = 4.25 > 3 \rightarrow Not OK \)
    - \( w_4(5) = 0.5 + 2 + 1.5 + 1.25 = 5.25 > 5 \rightarrow Not OK \)
    - \( w_4(6) = 0.5 + 2 + 3 + 1.25 = 6.75 > 6 \rightarrow Not OK \)
    - \( w_4(7) = 0.5 + 3 + 3 + 1.25 = 7.75 > 7 \rightarrow Not OK \)
    - \( w_5(9) = 0.5 + 3 + 3 + 2.5 = 9 \leq 9 \rightarrow OK \)
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

- Example (continued):

Graphical demonstration of time-demand analysis
Schedulability Testing
EDF Algorithm

- Task density:

\[ \text{density}_k = \frac{e_k}{\min(D_k, p_k)} \]

- A set of
  - independent,
  - periodic, and
  - preemptable

tasks can be feasibly scheduled by the EDF algorithm on one processor if the task set density is less or equal to 1:

\[ \sum_{k=1}^{n} \frac{e_k}{\min(D_k, p_k)} \leq 1 \]

Note: This is only a sufficient condition. Even if inequality is not satisfied, a feasible schedule might exist.
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Resources and Resource Access Control

Introduction

• If resources can only be used in a mutual exclusive manner, resource contentions occur that can lead to system failures.

• Effects of resource contentions:
  – Priority Inversions
  – Deadlocks
Resources and Resource Access Control

Effects of Resource Contention: Priority Inversion

- The phenomenon that a lower-priority task blocks a higher-priority task is called priority inversion.
Resources and Resource Access Control
Effects of Resource Contention: Uncontrolled Priority Inversion

• Uncontrolled (or Unbounded) Priority Inversion
A medium priority task can block a high priority task forever.

Uncontrolled priority inversion can only occur if the task set contains more than 2 tasks.
Resources and Resource Access Control
Effects of Resource Contention: Deadlock

• Consider two tasks $T_1$ and $T_2$ and two resources $R_1$ and $R_2$.
  – $T_1$ holds $R_1$, requests $R_2$
  – $T_2$ holds $R_2$, requests $R_1$

$\rightarrow$ Deadlock
Resources and Resource Access Control
Nonpreemptive Critical Section (NPCS) Protocol

- Simple way to control access to a resource is to schedule all critical sections nonpreemptively:
  
  If a task request a resource, it is always allocated the resource and executes with the highest priority.

  → This protocol is called the Nonpreemptive Critical Section (NPCS) protocol

- As no preemption takes place, no deadlock or priority inversion can occur!!!

- Shortcoming: Every task can be blocked by every lower-priority task, even if there is no resource conflict.
Resources and Resource Access Control
Basic Priority Inheritance Protocol (BPIP)

• The basic priority inheritance protocol (BPIP) prevents uncontrolled priority inversions but not deadlocks.

→ This is achieved by raising the current priority $\pi_l(t)$ of a lower-priority task to a higher (inherited) priority $\pi_h(t)$ of another task.

• BPIP rules:
  − *Scheduling Rule*: Ready tasks are scheduled preemptively in a priority-driven manner according to their *current* priorities. At the release time, the current priority $\pi(t)$ is equal to the assigned priority (the priority determined by the scheduling algorithm).
Resources and Resource Access Control
Basic Priority Inheritance Protocol (BPIP)

• BPIP rules (continued):
  
  – Allocation Rule: When a task $T$ requests a resource $R$ at time $t$,
    
    a) if $R$ is free, $R$ is allocated to $T$ until $T$ releases the resource, and
    b) if $R$ is not free, the request is denied and $T$ is blocked.
  
  – Priority-Inheritance Rule: When the requesting task $T$ becomes
    blocked, the task $T_i$ which blocks $T$ inherits the current priority of $T$
    until it releases the resource. At that time, the priority of $T_i$ returns to
    the value it had at the time when it acquired $R$. 
Resources and Resource Access Control

Basic Priority Inheritance Protocol (BPIP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T5 executes with priority 5</td>
</tr>
<tr>
<td>1</td>
<td>T5 is granted resource “black”</td>
</tr>
<tr>
<td>2</td>
<td>T4 released, preempts T5</td>
</tr>
<tr>
<td>3</td>
<td>T4 is granted resource “dotted”</td>
</tr>
<tr>
<td>4</td>
<td>T3 released, preempts T4</td>
</tr>
<tr>
<td>5</td>
<td>T2 released, preempts T3</td>
</tr>
<tr>
<td>6</td>
<td>T2 requests resource “black”, T5 inherits priority of T2 and executes</td>
</tr>
<tr>
<td>7</td>
<td>T1 released, preempts T5</td>
</tr>
</tbody>
</table>
### Resources and Resource Access Control

**Basic Priority Inheritance Protocol (BPIP), Example**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>T1 requests resource “dotted”, T4 inherits priority of T1</td>
</tr>
<tr>
<td>9</td>
<td>T4 requests resource “black”, T5 inherits priority and executes</td>
</tr>
<tr>
<td>11</td>
<td>T5 releases resource “black”, T4 continues</td>
</tr>
<tr>
<td>13</td>
<td>T4 releases resource “dotted”, T1 acquires resource “dotted” and continues</td>
</tr>
<tr>
<td>15</td>
<td>T1 completes, T2 is granted resource “black” and executes</td>
</tr>
<tr>
<td>17</td>
<td>T2 completes, afterwards T3, T4 and T5 execute and complete</td>
</tr>
</tbody>
</table>
Resources and Resource Access Control

Basic Priority Ceiling Protocol (BPCP)

- The basic priority ceiling protocol (BPCP) extends the BPIP to prevent deadlocks and to further reduce the blocking time.

- **Priority Ceiling**: The priority ceiling $\Pi(R_i)$ of a resource $R_i$ is the highest priority of all the tasks that require $R_i$.
  - Example (based on previous slide): $\Pi(B) = 2$, $\Pi(D) = 1$

- **Current Priority Ceiling (or simply ceiling)**: The ceiling $\hat{\Pi}(t)$ is equal to the highest priority ceiling of the resources currently in use. If all resources are free, the ceiling is equal to $\Omega$, a non-existing priority lower than any other priority.
  - Example (based on previous slide):
    - In (1,3], resource „black“ is used; hence the ceiling is 2
    - In (3,13], resource „dotted“ is used; hence the ceiling is 1
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

• BPCP rules:
  – *Scheduling Rule:*
    a) At its release time, the current task priority $\pi(t)$ is equal to its assigned priority.
    b) Every ready task is scheduled preemptively and in a priority-driven manner, depending on its current priority $\pi(t)$.
  – *Allocation rule:*
    Whenever a task $T$ requests a resource $R$ at time $t$, one of the following conditions occurs:
    a) $R$ is held by another task $\rightarrow T$ blocks
    b) $R$ is free
       a) If the priority $\pi(t)$ of $T$ is higher than the current priority ceiling, $R$ is allocated to $T$.
       b) If the priority of $T$ is *not* higher than the ceiling, $R$ is allocated to $T$ only if $T$ is holding the resource whose priority ceiling is equal to the ceiling; otherwise $T$ blocks.
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

- **BPCP rules:**
  
  - *Priority Inheritance Rule*: When $T$ becomes blocked, the task $T_i$ that blocks $T$ inherits the current priority of $T$. $T_i$ executes at its inherited priority until the time when it releases every resource whose priority ceiling is equal to or higher than the priority of $T$; at that time, the priority of $T_i$ returns to the value it had when it was granted the resource.
Resources and Resource Access Control

Basic Priority Ceiling Protocol (BPCP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T5 executes with priority 5</td>
</tr>
<tr>
<td>1</td>
<td>T5 is granted resource “black”</td>
</tr>
<tr>
<td>2</td>
<td>T4 released, preempts T5</td>
</tr>
<tr>
<td>3</td>
<td>T4 requests resource “dotted”, but the request is denied (priority of T4 lower than current ceiling). T5 inherits priority of T4 and executes at priority 4.</td>
</tr>
<tr>
<td>4</td>
<td>T3 released, preempts T5</td>
</tr>
<tr>
<td>5</td>
<td>T2 released, preempts T3</td>
</tr>
<tr>
<td>6</td>
<td>T2 requests resource “black” and becomes blocked by T5; T5 inherits priority 2</td>
</tr>
</tbody>
</table>
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>T1 becomes ready and preempts T5</td>
</tr>
<tr>
<td>8</td>
<td>T1 requests resource “dotted”; Priority of T1 higher than ceiling, resource request is granted</td>
</tr>
<tr>
<td>10</td>
<td>T3 and T5 are ready, T5 has higher priority (2) and executes</td>
</tr>
<tr>
<td>11</td>
<td>T5 releases “black” and its priority returns to 5; the ceiling drops to Ω; T2 unblocks, allocates „black“ and executes</td>
</tr>
<tr>
<td>14</td>
<td>J4 is granted “dotted” as its priority is higher than the ceiling</td>
</tr>
</tbody>
</table>
T4 requests “black”, which is free. The priority of T4 is lower than the ceiling, but T4 is holding the resource whose priority ceiling is equal to the current ceiling (“dotted”).

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>T4 requests “black”, which is free. The priority of T4 is lower than the ceiling, but T4 is holding the resource whose priority ceiling is equal to the current ceiling (“dotted”).</td>
</tr>
</tbody>
</table>