PROBLEM ONE Consider a system $S$ with input $x[n]$ and output $y[n]$ related according to the block diagram shown below.

The input is multiplied by $e^{-j\omega_0 n}$, and the product is passed through a stable LTI system with impulse response $h[n]$.

(a) Is the system $S$ linear? Justify your answer.

(b) Is the system $S$ time-invariant? Justify your answer.

(c) Is the system $S$ stable? Justify your answer.
(d) Specify a system $C$ such that the block diagram shown below represents an alternative way of expressing the input-output relationship of the system $S$.
(Note: The system $C$ does not have to be an LTI-system).

\[ x[n] \rightarrow h[n]e^{j\omega_0 n} \rightarrow C \rightarrow y[n] \]
Problem Two: Signal \( x[n] \) and \( y[n] \) shown in the Figure 1.1 below are the input and corresponding output for an LTI system.

\[ x[n] \]
\[ \begin{array}{c}
0 \\
1 \\
-1 \\
n
\end{array} \]

\[ y[n] \]
\[ \begin{array}{c}
0 \\
1 \\
-1 \\
n
\end{array} \]

Figure 1.1

(a) Sketch \( y_2[n] \), the response of the system to the sequence \( x_2[n] \) in Figure 1.2.

\[ x_2[n] \]
\[ \begin{array}{c}
0 \\
1 \\
-1 \\
n
\end{array} \]

Figure 1.2

(b) Find the impulse response \( h[n] \) of this LTI system.
Keys

P1.
(a) Yes. It's linear.
(b) No. It's time-varying.
(c) Yes. It's stable.
(d)

\[ C = e^{-j\omega_n} \]

P2.
(a)

\[ y_2[n] \]

\[ n \]

0 1 2 7

-2 -1
\[ h[n] \]