Information Maximising Optimal Sensor Placement Robust Against Variations of Traffic Demand Based on Importance of Nodes

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\textbf{Abstract}—This paper defines a measure of importance for a node (intersection) in a transportation network, based on its topology and traffic demand. Consequentially the measure is used in order to solve the sensor placement problem by maximising the information gain in terms of users’ routing choices by sensing the most uncertain areas in the system. It is demonstrated that utilising the proposed strategy makes the performance robust against short and long term variations of traffic patterns. Finally, a method for finding the optimal number of sensors to be installed in a city is proposed. It models and maximises the utility stemming from the trade-off between cost, performance, robustness and reliability of the sensor placement problem solution.

\section{I. INTRODUCTION}

Identifying the most important modules or elements of a complex system is a problem that is of great interest to engineers and researchers. Its most significant entities or subsystems are, depending on the system, either cautiously monitored, robustly controlled, or rigorously studied in order to gain deeper understanding of the system's dynamics.

In the case of transportation systems, “important” parts of the network are usually sensed in order to get information about the overall traffic state. Engineers go even further by trying to change and control traffic parameters at such locations by planning new infrastructure developments \cite{1}, control strategies \cite{2}, novel policies \cite{3}, etc.

The aim of sensing traffic has been mostly in order to determine the flows in a city. The problem of optimal placement of counting sensors in order to estimate an Origin - Destination (OD) matrix has been around for more than four decades \cite{4}. Knowing the OD matrix, the flows can be extracted and knowing the flows, the traverse times on the respective roads can also be evaluated, thus providing aggregated information about the traffic situation.

Given the increased pace of introduction of new technologies to the market and growing availability of computing power, traffic sensing and city planning are getting more interdependent and strongly connected. There are methods that use sensed data in real time in order to apply changes to the traffic system \cite{5}. Therefore, sensors may not be placed with the sole reason to observe traffic. Smart cities use their sensors’ data streams in order to optimise their performance in real time. With the increased number of sensor types such as plate scanning, velocity measuring, emissions measuring, etc. and their reduced error rate, it is now a matter of great importance to shift the sensor placement problem toward a more active goal. The information stream coming out from the sensors should be utilised by control algorithms or long term planning strategies in order to “actively” sense the traffic by controlling it at the same time.

Fundamentally, sensors are put in such positions so that they maximize the information gain. Naturally, the locations that need to be sensed in order to maximize the information gain are the ones of higher importance. In other words, the chosen locations to be sensed are usually the ones that we are most uncertain about with respect to a predefined information measure. In case the uncertainty of both nodes is of equal magnitude, the one that is more used is of greater importance. Therefore, the uncertainty of users’ choices at every node should be weighted by the number of users that utilise it. Depending on the definition of information the placement problem can take different forms.

In most cases, information is considered to be a characteristic of the link, like throughput or flow velocity. If, however, we want to find the intrinsically important locations, we need to look for the places where the choices, that lead to those characteristics, are made. Average flows, velocities, densities on road segments are perceived as the factors that describe traffic conditions. The main factor that determines all those, however, is the routing choices that the users make. Routing choices can account for a significant difference of traffic performance as shown in \cite{6}, where a 20\% improvement of average travel time for the whole system was demonstrated by just optimally selecting the paths of the users. A novel and more efficient approach for sensing traffic would then be to try and maximise the information about the routing choices of commuters rather than the flows, speeds and densities on the roads.

By knowing the routing choices of drivers at key intersections, a detailed map of the specific flows on the links can be inferred, which is what usually sensor placement strategies aim at. In addition to this, however, information about where the flows come from is also available. Therefore, by gaining information about the routing choices, both the link and path flows can be simultaneously approximated. Furthermore, by examining differences between sensed flows and predicted ones from the routing choices, the OD matrix can also be

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estimated. Therefore, routing choices stand as the source of information that determines the usually addressed problems of estimating OD matrix, path flows and link flows.

Intersections are the places where commuters make choices, and what is sensed at the connecting road segments are just the consequences of those choices. Therefore, instead of examining links in a traffic network, a more topologically central approach would be to examine nodes (intersections) instead. It is important to note that the uncertainties and the dynamics of each node are expected to be weakly correlated with respect to their spatial connections. Contrary to the speed, flow, etc. measures of road segments, which are usually correlated in case of physical proximity, the uncertainty at nodes tends not to have a distance dependent correlation. This key difference diminishes the need for a combinatorial mutual information optimisation approach of high complexity in order to find the optimal sensor placement, because there is intrinsically no redundant information in the sensing network.

Sensing a node or a group of nodes representing an intersection in a road network boils down to tracking the decisions of the users that pass through it. This task can be performed by placing plate scanning sensors on the roads that lead to the intersection and out of it. This sensing architecture allows us to both evaluate the turning choices of the commuters as well as still collect information about flows on the separate links. More than that, since in order to properly “sense” a node we only need the flow along the edges, sensors can be placed at any position on the edge. Thus, if positioned at a midpoint along the edge, the sensors will also be able to collect information about the cruising speed along it, therefore maximising the information input.

Another pressing matter that has been ignored in the past is the robustness of a sensor placement solution. Usually, robustness is understood as the error rate or redundancy of a particular sensor placement. In this work, however, we look at robustness from a different angle. Due to the fact that sensors are quite expensive and their installation consumes both time and resources, we want to minimize the need to move the sensors around (if at all possible) after they are once installed. In this sense, robustness of a sensor placement can be defined as the property of the set of locations to stay important when the traffic demand conditions in the system are changed. Such changes may include short term changes in the OD matrix such as daily variations of traffic (evening rush hour vs. morning rush hour or weekday against weekend) and also long term changes in the network demand such as people moving around the city and changing living districts and jobs, building of new living complexes or business centres, etc. Such changes may severely alter the situation for a given sensor placement and thus make the investment for their installation obsolete. The robustness of a planned sensor network against variations in the traffic demand is of great importance, especially for constantly evolving large cities.

In a more fundamental aspect, traffic is mainly determined by two factors: traffic demand (where and when do people want to go) and transportation network topology (the medium that allows the commuters to move). The existing measures governed mostly by the demand are link flows, path flows, link average speed, etc. Measures connected to the topology are centrality, heterogeneity, entropy, etc. There have been previous efforts to define entropy (uncertainty) of a node or a link but only in a purely topological sense [7]. We strongly believe in the need of employing the information contained in the OD matrix, namely the traffic demand, as well in order to come up with a more useful definition and measure of the uncertainty of a node and the importance of it being sensed. In this study we define the entropy of a node and consequently its importance by using both information about the traffic demand and topological information about the network, which surely gives a better overview than basing the definition on just one of them. The information from both sources must be entangled since they are actively affecting each other. In this way a single measure that represents all the available information can be defined.

The main contributions of this work are:

- Definition of entropy of a network and importance of nodes.
- Study on the robustness of the measure against changes in the OD matrix.
- Design of methods for finding the most robust optimal sensor placement against short and long term variations
- Design of a method for finding the optimal number of sensors to be placed in a given network.

II. LITERATURE REVIEW

Determining the importance of locations in traffic networks is crucial. There are two main branches of research that are interested in locating central spots in a network. The first one is traffic sensing. In most sensor placement problems, the set of locations to be sensed is chosen so that, the resulting synthesis of data is the most informative, which boils down to sensing the important locations that describe the traffic demand. The other area is complex networks research. Importance is then defined and studied in a purely topological sense by examining the transportation network without considering any traffic demands. This review will cover both areas with an emphasis on the sensor placement studies.

There are many attempts to find optimal sensor placement in order estimate an important traffic characteristic. One of the most comprehensive surveys [8] discusses and summarizes existing sensor location problems. It defines the traffic sensing problems into categories depending on sensor types (AVI sensors, counters etc.), prior information and flows of interest (link flows, route flows, OD flows). The optimisation problems are divided into two categories: Flow observability problems and flow estimation problems. Moreover, it describes different rules for optimisation and analyses methods such as flow intercepting, demand intercepting, independence of traffic counts (mutual information). This work is valuable because it summarizes and categorizes the various approaches by providing a unifying picture of existing strategies.

One of the most standard traffic characteristic to be observed is the OD matrix. Estimating it from sensor data has become a central problem. In [9] the sensor location problem for OD matrix estimation is defined and a solution is suggested.
The study deals with counting sensors, while other studies also include the possibility of using (AVI) Automated Vehicle Identification readers, which are more informative since they also collect information about the identity of the car, which allows for easier tracking and therefore path estimation [10]. In [11] both types of sensors are used in a method that places counting sensors and AVI readers to maximize the expected information gain for an OD demand estimation problem. It also takes into consideration uncertainty in historical demand information. A technique for calculating the optimal number and locations of plate scanning sensors for a given OD matrix is also presented in [12]. Those approaches are centred around the goal of estimating the OD matrix. In most of the cases they are applied on artificial networks as a proof of concept, however their high complexity might turn into a disadvantage if one tries to apply such a strategy for a real life large city. Therefore there is a need for sensor placement method that is less computationally intensive so that it is practically applicable.

Once the locations of sensors are fixed one might use a linear approximation technique in order to estimate the OD pairs using traffic counts offline such as the one described in [13]. In case plate scanning sensors are used a method for path reconstruction from such type of data can be used as in [10]. In [14] methods for extracting information from sensors data in order to estimate travel times are discussed, while also looking at sensor failure probabilities. Furthermore, due to the heterogeneous nature of collected data, information demands and the limited storage capacity of road side sensors, a maximum content dissemination strategy must be employed as well as done in [15]. Another issue that must be taken into account once the road segments that need to be sensed are determined is the feasibility of positioning a sensor there. More precisely, it must be verified if the sensor will be able to handle the volume in the sense of contact time and contact rate or more sensors must be placed at the same road segment in order to increase the accuracy of the extracted data. A rigorous analysis of these problems can be found in [16].

There are more universal approaches for choosing the most important locations to be sensed, which are based on maximizing information gain. There are information theoretic techniques such as [17], where a non-myopic strategy is used to find the most informative locations for sensors, [18] where a Kalman filtering structure is employed in order to solve a traverse time prediction problem via optimally placing sensors, and [19] where a method for target localization and tracking is presented, which computes the posterior target location distribution minimizing its entropy. Furthermore, in [20] the spatial and temporal correlation between the flows are used in order to feed an ant colony optimisation algorithm that finds optimal sensor locations.

Information theoretic approaches, however, may vary among each other. In [21] traffic phenomena are modelled as Gaussian processes. They discuss maximizing entropy for sensor locations and also mutual information between the locations and demonstrate that the mutual information approach performs better for certain type of scenarios. Moreover the method is extended to find robust placement against failures of sensors and uncertainties in the model and uses real world data sets. This is a generic method that can be applied to different types of sensors. It locates the most representative links in the network that reduce the uncertainty about the unobserved links. There is, however, no method that is able to determine the most important links in the sense of locations where users makes the choices that are later observed at the representative links.

With the advancement of technology some type of sensors now can be mobile instead of static, while granting better coverage. In [22] a mobile traffic surveillance method is presented. A routing problem is defined such that it computes the optimal paths for the mobile sensors and show that in most cases it performs better than a static network. Mobile sensors provide several advantages such as bigger area of coverage, adaptability to changes in traffic patterns and are the better approach when there is no prior knowledge about the system. In [23] a strategy for a sensor placement for monitoring mass objects is described. By allowing the sensors to be mobile the sensing network can self-organize in order to achieve better coverage. An expectation-maximization algorithm is used in order to update the distributions of objects, which are then used to implement an adaptive sensor placement strategy for the desired tracking task. On the other hand, static sensor placements are easier to implement, cheaper to maintain and due to their static nature are able to use technologies that are more complex and precise. One more advantage of static sensor networks is that they are able to provide a better “instantaneous” picture of the traffic situation in the sense that mobile sensors collect samples that vary both spatially and temporally, while static sensors collect much larger number of samples for exactly the same time period. Especially in rush hour conditions with fast changing traffic patterns such temporal stability is valuable for a more precise estimation of traffic characteristics.

In the findings of [24] the authors demonstrate using traffic indicators that importance of road segments is mainly determined by the network structure and the flows. Even though, this statement is clearly known there is still no indicator of importance of road segments that fully utilizes the flow information and the topological properties of the network. As it can be seen most methods to determine important locations for sensor placement are based mostly on the flows; while in a separate part of literature people look at purely topological properties of transportation graphs.

Important locations can be determined based solely on the topology of a network. Some efforts deal with identifying critical links using a network robustness index based on link flows, link capacity and network topology as in [25]. In [26] the most vital links or nodes are defined as the first $n$ links or nodes whose removal will lead to the biggest increase in average shortest path distance. While in [27] the importance of roads is simply defined to be proportional to the traffic load on them, in [28] three measures of centrality for a street are suggested: closeness, betweenness and straightness and their correlation to various economic activities in the respective areas are examined.

Moreover, the network itself can have some properties that
are usually based on the structure of the system and not on local properties of its elements. In [29] the development of the Swiss road and railway network during the second half of the 20th century is investigated. It is observed that the spatial structure of transportation networks is very specific, which makes it hard to analyse using methods developed for complex networks. In [30] existing measures of heterogeneity, connectivity, accessibility, and interconnectivity are reviewed and three supplemental measures are proposed, including measures of entropy, connection patterns, and continuity. Entropy is also used in order to determine the heterogeneity of the network regarding a chosen parameter.

The topology of a network holds an enormous amount of information. It may provide insights into the structure of the roads (transportation networks are organized hierarchically as shown in [31]). In [32] they measure the efficiency and accessibility in Paris and London based on the network connectedness. Moreover, this information can be utilized in order to reconstruct user’s trajectories from GPS signals as in [33]. There is also a family of graph measures based on entropy that are rigorously summarized in the survey [7]. It includes some measures from chemical structural analysis and social network analysis. The survey examines the overall connectedness of graphs such as the topological information content and the entropy of the weights of the edges. A measure of local features’ such as entropy of nodes is defined as well, based on length of links connected to it. The centrality measure of links is also defined. Most of the measures deal with evaluating the information content in the graph itself. Those measures are highly uncorrelated, which means that they capture different aspects of graphs, so the proper measure should be chosen for each specific task.

Once a measure of importance is defined and the most informative locations are chosen, there is one more aspect that needs to be examined. The robustness of those choices depends on the evolution of both the topology of the network and on the evolution of the OD matrix as well. Those two factors are naturally also highly interdependent. In [34] the evolution of the topology of networks is observed. A high degree of self-organization and spontaneous organization of hierarchies is observed in the city of Indiana. Also variations in the relative importance of parts of the network are observed. In [35] the evolution over 200 years of a North Milan road network is observed. Two main processes can explain the developments that occur. Densification of the road network around the main roads and emergence of new roads as a results of urbanisation. An evaluation of the robustness against such type of long term network evolution for any type of sensor placement is lacking at the moment.

In order to analyse traffic and plan for its surveillance one needs a model. Dynamic traffic assignment models such as the one described in [36] need a dynamic network load model and routing choices of users model, which basically means that they need the OD matrix combined with a routing model such as in [37] based on stochastic conditions. Although patterns seem not to vary excessively as observed in [38]. It is shown that daily traffic is highly predictable and that there exist regular patterns that can be exploited. This stability of choices made by traffic participants together with network topology also leads to traffic concentration on mainly a few links of the network as shown in [39].

### III. Measuring Importance of Nodes

In this section the measure of importance of nodes is introduced. We define a node as important if many users pass through it and we are uncertain about the choices they make. In order to get the uncertainty an entropy measure at the node is needed. Following that we simply weigh this measure by the throughput of users. In this way we can measure how much this node adds to the overall uncertainty of the road network given an OD matrix. Let us introduce some notation that will be used throughout the paper first:

- \( N_{ij} \) - number of cars that moves from node \( i \) to node \( j \)
- \( P_l \) - the path of the \( l \)-th user
- \( f_{ij}(l) \) - function that is one if the sequence of nodes \( ij \) is the that path of user \( l \)
- \( A \) - a set containing all the users
- \( p_{ij} \) - probability that an user that is at node \( i \) will continue on to node \( j \)
- \( S_i \) - set of nodes that are successors to node \( i \)
- \( H_i \) - entropy of node \( i \)
- \( I_i \) - importance of node \( i \)
- \( T \) - number of regions the day is split into
- \( N_{ij}^{t} \) - number of cars that pass sequentially through node \( i \) and \( j \) during time period \( t \)
- \( H_i^t \) - entropy of node \( i \) during time period \( t \)
- \( I_i^t \) - importance of node \( i \) during time period \( t \)
- \( I_i^d \) - the overall importance of node \( i \) for a degree of perturbation \( d \)
- \( R \) - total reduced entropy
- \( L \) - a set of sensor locations
- \( L^d \) - locally optimal sensor placement for a degree of perturbation \( d \)
- \( L_o \) - globally optimal robust sensor placement
- \( Var_d[I_i^t] \) - the mathematical variance of \( I_i \) across all possible values of the \( d \) coefficient
- \( E_d[I_i^t] \) - the mathematical expectation of \( I_i \) across all possible values of the \( d \) coefficient
- \( g^d \) - a function that takes as argument a set of sensor locations
- \( V_{L_o} \) - variation level of the importance values of sensor placement \( L_o \)
- \( M_{L_o}^i \) - percentage of mismatched sensors between locally optimal placement \( L^d \) and globally optimal robust placement \( L_o \)
- \( M_{L_o} \) - the overall percentage of mismatched sensors for all degrees of perturbation
- \( Q_{L_o} \) - performance measure of robust optimal solution \( L_o \)
- \( K_{L_o} \) - cost of installing solution \( L_o \)
- \( U_{L_o} \) - utility function value of solution \( L_o \)

Shannon’s entropy is calculated using the transition probabilities between the states of the system. Let us assume that
the state of an user is its current link. The set of possible transitions from this state represents the set of actions of the user turning on any of the links that are successors of the current link. The entropy of the node connecting those links is calculated using this information.

The following are the steps taken in order to calculate the importance of a node:

1) **Calculate turning probabilities:**

Let \( N_{ij} \) be the number of cars that pass through the \( i \)-th node and after that through the \( j \)-th node, where node \( j \) is a successor of node \( i \) in the directed graph describing the road network, and let \( P_l \) be the path of the \( l \)-th user. Then let the function \( f_{ij}(l) \):

\[
f_{ij}(P_l) = \begin{cases} 
1 & \text{if nodes } ij \text{ are in } P_l \\
0 & \text{otherwise}
\end{cases} \tag{1}
\]

Then:

\[
N_{ij} = \sum_{l=1}^{\mid A \mid} f^l_{ij}(P_l) \tag{2}
\]

, where \( \mid A \mid \) is the number of users.

Let \( p_{ij} \) be the probability that an user at node \( i \) continues to node \( j \)

Let \( S_i \) be the set of nodes that are successors of node \( i \). Then we can define the turning probability as the ratio between the number of cars that pass through node \( i \) and then proceed to node \( j \) and the total number of cars that pass through node \( i \):

\[
p_{ij} = \frac{N_{ij}}{\sum_{k \in S_i} N_{ik}} \tag{3}
\]

2) **Calculate the entropy at every node:**

The entropy of a node \( i \), \( H_i \), is calculated using Shannon’s entropy definition. A state is represented as the current link an user is on and the transition probabilities are the turning probabilities from this node to its successors. Then the entropy becomes:

\[
H_i = -\sum_{j \in S_i} p_{ij} \log p_{ij} \tag{4}
\]

3) **Weight the entropy of every node with the number of users that pass through it:**

In order to differentiate between nodes that have a high entropy value that have high and low traffic throughput, we weight the entropy of every node by the number of users utilising it. The importance of node \( i \) is defined as:

\[
I_i = H_i \sum_{j \in S_i} N_{ij} \tag{5}
\]

![Fig. 1: Illustration of a suggested positioning strategy for sensors for different types of intersections.](image)

Fig. 1: Illustration of a suggested positioning strategy for sensors for different types of intersections. On Fig. 1a the graph representation of an intersection with all allowed turns is presented. As it can be observed this scenario can be represented with a single node connecting all the edges leading to the intersection and leading out of it. On Fig. 1b the green circles represent plate scanning sensors that need to be installed in order to be able to obtain the necessary information to calculate all the turning probabilities. It should be noted that this is the simplest possible case and that the sensors positioned at the intersection do not even need to collect the id of the passing vehicles in order to evaluate the turning probabilities. On Fig. 1c we have depicted a slightly more complicated case where U-turns are not allowed. In this case 8 separate nodes are needed in order to represent the intersection using an unidirectional graph. As observed in Fig. 1d all edges from and to the intersection need to be observed by the plate scanning sensors in order to extract the needed information to calculate the turning probabilities and the entropy of all nodes.

Although the main goal of this work is not to design the physical architecture needed to sense an intersection, we have provided an example architecture on Fig. 1. Since sensing a certain intersection might require several sensors placed in close proximity it is important to comment on the redundancy of the positions of sensors.

When deploying sensors in order to maximise information about flows or velocities using common methodologies, the measured values are usually highly correlated in space due to the fact that what is being sampled is in fact continuous sequence of road segments that belong to the same road. In the case of sensing, based on the proposed importance of a node measure, the main value that determines the priority of sensing a certain intersection is the collection of turning probabilities at it. We believe that it is trivial to observe that those are not correlated in space.

Let us assume that we have an intersection between two main roads. The importance value for this intersection is likely to be high and therefore a group of sensors should be placed there. It is not likely that there is such an intersection in close proximity to this one, since this would be considered as an inefficient design and even if there is another major intersec-
tion in close proximity, this would be a topological peculiarity of the road network rather than an intrinsic property of spatial correlation as in the case of sensor placement strategies that maximise information about the flows or average speeds.

Therefore, sensor placement based on importance values of intersections is intrinsically not prone to redundancy of sensor positions. If a situation occurs where two neighbouring intersections both possess a high importance value it is vital to understand that this is not a redundancy issue and in fact both intersection should be sensed in this case since the information acquired from each of them is different. Naturally, the edges connecting the two intersections should not be sensed twice.

An example of calculating turning probabilities, entropy and importance of nodes is given in Fig. 2. Furthermore the exact positions of the sensors in the simple example network are shown depending on the number of sensors to be put.

![Diagram of Example Network](image)

**Fig. 2:** Diagram providing an example of calculating importance of nodes. The first thing to do is to calculate the entropy of every node. Nodes 3, 4 and 5 have only one successor, which means that there is only one turning probability with value 1, which means that the entropy and therefore the importance of those nodes is 0. As it can be seen on the graphs, no choices are being made at those nodes, therefore they have a small importance value. The entropy of node 1 is calculated using Eq. 2 is $H_1 = 0.3$ and the importance from Eq.3 is $I_1 = 3$. Similarly for the other nodes $H_2 = 0.41$ and $I_2 = 2.05$. Therefore, the nodes that are of interest and in this case have non-zero values are nodes 1 and 2. In case sensors for one intersection are available the most important node (1) is sensed. The precise positions of the sensors are on the links $L12$ and $L13$, which ensure complete knowledge of the choices made at the intersection. In case one more intersection can be sensed, obviously it should be intersection 2 and the green sensors represent the additional links that should be sensed. Since there already is a sensor placed at $L12$, we only have to add sensors on $L23$ and $L24$ in order to gain complete information about the routing choices at node 2. Since the entropy of all other nodes is 0 after the placement of the red and green sensors it is guaranteed that we have full information about the network.

Changing traffic demands over the course of a day results in the importance value of a node changing as well. Some nodes may experience high importance values during morning rush hour while having lower values during the evening. In case sensors are placed at nodes, whose importance value varies significantly throughout the day, they cannot be moved if some other nodes become more important. This is the reason why we need to find out the nodes that overall, have the biggest importance values across the day.

Since it is also important to study the daily variation of importance let us examine the notation describing splitting the day into time-of-day (TOD) intervals:

- $N_{ij}^t$ - the number of users that go from node $i$ to node $j$ in period $t$
- $H_i^t$ - the entropy of node $i$ during period $t$
- $I_i^t$ - the importance of node $i$ during period $t$

Next step is to come up with an importance value representative for the whole day. Some regions of the day are of less interest than others simply because the amount of information that can be extracted is smaller. Typically, the factor that plays the largest role in this case is the amount of traffic. Therefore, we compute the total importance of a node for the whole day, using a weighted average of importance values of the node for different regions of the day. The weight function is governed by the number of users that pass through the node during the respective time region. Then, we can define the overall daily importance of a node as:

$$I_i = \sum_{t=1}^{T} I_i^t \frac{\sum_{k \in S_i} N_{ik}^t}{\sum_{k \in S} N_{ik}}$$

The second term in the sum is simply the number of cars that pass through the node throughout time region $t$ over the total number of cars that pass through the whole day and $T$ is the number of regions the day is split into. This definition of overall importance puts an emphasis on the nodes that are interesting during the important parts of the day. This weighting is included in order to avoid high importance values throughout periods of time where the node is not being utilised.

### IV. Achieving Robustness Against Changes in the OD Matrix

In reality, apart from the changes in the OD matrix over the course of the day, there is another process that alters the traffic demand in a less intense and more gradual way. This process is a result of long term changes to both the user population and the city structure. In order to demonstrate that our method is robust against such type of variations we implement a generic way to “alter” or “perturb” the traffic demand.

#### A. Methodology for altering the OD matrix

Let every user have a list of itineraries, which is composed of separate trips. Every trip has an origin, destination and start time. In most cases an user takes two trips per day: from home to work in the morning and from work to home in the evening. Let us take two users. We assume that the first origin
and the last destination in the itinerary of those users is their place of residence. Then we exchange those locations, as if the first user now lives in the home of the second user and vice versa. We do this for a predetermined percentage of all the users. We call this percentage the degree of disturbance. By executing this strategy the number of people starting from or arriving at all the regions is not changed. This means that the intensity of people starting from any region is not changed and the intensity of people arriving at those regions is also not altered. The factor that is perturbed is precisely the OD matrix, since only the intensities of the connections between origins and destinations are varied.

This procedure is visualised in Fig. 3

![Diagram illustrating the exchange of origins or living locations of two users. Both users still have the same work locations however they switch their homes. In this way the OD pairs intensity is changed.](image)

**B. Strategy for robust placement**

In order to find locations that are optimal for performance and robust against variations in the OD matrix we have to find a measure that represents the importance of a set of nodes for different degrees of perturbation. This is the overall importance of the chosen locations. Every node \( i \) has an importance measure \( \bar{I}_i \). We assume that we can take a given number of sensors from all possible locations and then we calculate the total reduced entropy in the network which is:

\[
R = \sum_{i=1}^{\vert L \vert} \bar{I}_i 
\]  

(7)

For every different degree of perturbation every node has a calculated importance value \( \bar{I}^d_i \) where \( d \) is the degree of perturbation.

Let the resulting reduced entropy from a set of locations \( L \) for different degrees of perturbation \( d \) be calculated by the function \( g^d(L) = R \), let the optimal placement for a given degree of perturbation \( d \) be \( L^d \) and \( g^d(L^d) = R^d \).

We are looking for an optimal placement \( L_o \) that maximizes the reduced entropy relative to the local maximum across the various perturbations:

\[
\max_{L_o} \sum_d g^d(L_o) / g^d(L^d) 
\]  

(8)

**C. Strategy for finding the optimal number of sensors**

There are four aspects that should be taken into account when designing a utility function to be maximised in order to find the optimal number of sensors.

1) **The variation of the importance value across the perturbations:**

Every node has a different importance value across the perturbations \( \bar{I}^d_i \). We want to evaluate the degree of variation so that we can locate globally important nodes rather than nodes that have just one high importance value among various degrees of perturbation. In order to do that, we calculate the variance for every node \( i \) across different degrees of perturbations \( d \): \( Var_d[\bar{I}^d_i] \). We normalise it by the average value across the perturbations so that this measure is comparable to others:

\[
\frac{Var_d[\bar{I}^d_i]}{E_d[\bar{I}_i]} 
\]  

(9)

In order to evaluate the total variation level of the importance for a sensor placement we calculate the average of the scaled variances for all chosen locations:

\[
V_{L_o} = E_i \left[ \frac{Var_d[\bar{I}^d_i]}{E_d[\bar{I}_i]} \right] 
\]  

(10)

We can vary the number of nodes to be included in the set of optimal locations \( L_o \); this can also be referred to as its cardinality: \( \vert L_o \vert \). The goal is to minimise the variation of importance of the same node across the degrees of perturbation in order to ensure robustness of the placement.

2) **The percentage of mismatched sensors:**

We define \( M^d_{L_o} \) as the percentage of sensors that are mismatched between the optimal sensor placement for a certain degree of perturbation \( L^d \) for a given number of sensors, and the robust optimal solution \( L_o \). This is basically the cardinality of the difference between the two sets divided by the cardinality of the set:

\[
M^d_{L_o} = \frac{\vert L_o \setminus L^d \vert}{\vert L_o \vert} 
\]  

(11)

Then the overall percentage of mismatched sensors is just the average of this measure across all degrees of perturbation:

\[
M_{L_o} = E_d[M^d_{L_o}] 
\]  

(12)
This is a measure of distance between the optimal solution for \( d \) and the robust optimal solution for all degrees of perturbation. It can also be understood as a value signifying the percentage of sensors that need to be moved in order to reach the local optimal solution. This measure should be minimised if we want to ensure robustness of the placement. In other words, the sensor locations should be as universal as possible.

3) **Performance measure of the robust optimal solution compared to the local optimal solutions:**

This measure is used to describe how close is the robust optimal solution to perfectly match the locally optimal solutions.

\[
Q_{L_o} = \sum_d \frac{g^{d}(L_o)}{g^{d}(L_o)}
\]  

(13)

This measure should be maximised since we aim for maximum performance.

4) **Cost of sensors:**

We also include a function that punishes high number of sensors. For simplicity we just use a linear function that grows with the increase in number of sensors:

\[
K_{L_o} = \alpha |L_o|
\]  

(14)

The utility function that needs to be maximised subject to the number of sensors or the cardinality of the set \( L_o \) then becomes:

\[
\max_U \sum_{L_o} w_1 Q_{L_o} - w_2 V_{L_o} - w_3 K_{L_o} - w_4 M_{L_o},
\]  

\[
\text{where } \sum_{i=1}^{4} w_i = 1
\]  

(15)

All the separate functions are scaled to assume values between 0 and 1, however depending on the designer’s choice some measures can be given more weight by varying \( w_{1-4} \).

On Fig. IV-C we can see all the separate functions and the utility function that determines the optimal sensor number.

**V. Case Study: Singapore**

A. **Modelling the Traffic in Singapore**

In order to collect all the data needed by the methodologies described in the previous sections, the traffic in a specific realistic scenario should be modelled and simulated. The procedure is as follows:

- **Generate users:**
  
  The data source used for user generation is the Household Interview Travel Survey (HITS) of Singapore done in 2012. It contains information of a representative sample of people (about 1 % of the population) that state their daily travel patterns. For every person that participated we can extract the information about all trips that are made on a daily basis consisting of origin point, destination point and starting time of the trip. We use those “sample” points in order estimate an origin destination matrix for the city and its variation in time. For every user that we want to generate, we choose one of our “samples” at random and add Gaussian noise to it, both in space and time, meaning that we slightly vary the starting position and the starting time of the user in order to get a more realistic homogeneous distribution that is still based on the real world data that we have. The total number of users that we generate is around 300,000 with starting times of trips varying throughout the day. Most of the users have two trips in their itineraries, one in the morning (between 6:30 and 9:30) and one in the evening (between 5:30 and 7:30), however there is traffic in the city throughout the whole day.

- **Calculate the routes of the users:**
  
  After we have the generated users, we use the origins and destinations in their itineraries to calculate their actual routes. The weights on the routing graph that we use represent the time it would take the user to traverse the link with the maximum allowed speed on this link. This assumes that the commuters would prefer the fastest path. Our model does not allow for re-routing of the users based on congestion factors. After the routes are computed, we have the sequence of links that every user passes through, which is then used in order to calculate the turning probabilities for the node entropy calculation. The day is split into 48 pieces consisting of 30 minute intervals. In order to calculate the entropy values of every node for the respective time periods, we use the starting time of the trip as a time-stamp of the whole route and in this way organize the passages through the links in time.

B. **Importance Analysis**

In this section we demonstrate the functioning of the described methodologies in a case study of Singapore. We start with calculating the entropies of every node of the network within each of the time periods that the day is split into. A video of the evolution of the importance of nodes in the city throughout a full day will be available at http://ieeexplore.ieee.org.

Following this we apply the robustness against daily variations technique. This allows us to get the overall daily importances of the nodes and use them to find the optimal sensors placement as described in section III. Fig. 4 shows the Singapore road network and the importance of nodes. It can be observed that the sensors cover the city well with accents on the central business district (south central part), the highway intersections and intersection of highways with other large roads. Moreover, there are plenty of sensors in the residential areas (east and north central), which, however, have lower importance values due to the smaller number of cars that go through those intersections.

Next, we try to simulate change in traffic demand as explained section IV-A. Fig. 5 visualises the results of applying the change. Since it is not practical to visualise all OD pairs, in our visualisation we show explicitly the intensities of OD pairs that have as origin the university area around the Nanyang Technical University (NTU) in the western part of the city. We can observe the change in the destinations intensities as...
people increase their trips to the east part of the city while reducing the trips that stay within the western part.

The following step is finding the optimal sensor placement for Singapore that is robust against such type of variations in the OD matrix as described in section IV-B. In order to evaluate the performance of the robust placement we run the following experiment:

1) For each degree of perturbation run a set of 10 simulations in order to get an averaged value for all the required parameters. The number of simulations is determined so that the degree of variation is below a certain threshold as described in [40].

2) Using the simulation outputs, calculate the turning probabilities, entropies, and importance of all nodes.

3) Find the optimal placement of sensors for every degree of perturbation.

4) Using the optimisation strategy described above, calculate a robust sensor placement.

5) Compare the performance of the robust sensor placement to the performance of the locally optimal (in the sense of perturbation degree) sensor placements. The performance in this case is the ratio between the total reduced entropy $R$ of the robust placement to the total reduced entropy of the locally optimal placements.

In Fig. 6 we can see a comparison of the performance of the optimally robust method versus the locally optimal solutions for sensor placement.

Following this, we calculate the performance of those sets of locations for other degrees of perturbations. For example the blue line on Fig. 6 represents the optimal sensor placement for the original OD matrix. Naturally, since the sensor placement was made based on the traffic patterns in this scenario, the performance is 100%. We can then see that the performance of this sensor placement if the traffic is governed by the OD matrix perturbed by 5% decreases. The more we perturb the traffic demand, the more the performance of the optimal sensor placement calculated from the original OD matrix decreases. The goal of the method is to achieve robustness in the sense that the performance stays consistently high as we vary the OD
matrix. We have also plotted the performance of our robust sensor placement solution as the black dotted line. It can be observed that the robust solution does not vary that much when the OD matrix is perturbed and is performing better than the rest. It can be concluded that the performance of optimal sensor placements calculated from a specific OD matrix vary more than the performance of the sensor placement strategy that uses not just one OD matrix but rather a set of perturbed variations of it. Moreover, it can be observed that the defined measure of overall importance proves to have very little degree of variation, which makes it a suitable candidate for a robust measure of importance of nodes.

Finally, we compute the optimal number of sensors to be placed in Singapore as described in section IV-C. For the sake of simplicity let all the discussed factors be equally important. In Fig. 7 the functions related to the process of finding the sensor count are plotted. On the last sub-graph we can see the utility function whose maximum corresponds to the optimal number of sensors to be installed. We can see that in the case of Singapore this number is 582. Surely, if some factors are more important than others, they can be weighted differently and this will affect the optimal number of sensors.

VI. CONCLUSION

In this paper we have pointed out the need for an importance measure that is able to combine and refine the information about the traffic demand contained in the OD matrix and the information about the topology of the network. In this way one can point to the locations in the network that hold the biggest amount of uncertainty related to drivers’ routing choices; this we believe is a crucial underlying factor that determines traffic conditions in a network. We have discussed that nodes should be examined instead of links since the intersections are the places were decisions are made and the roads are the locations were the results of those choices are observed.

We have defined a measure of importance that satisfies the aforementioned conditions using information theory. More precisely, the measure is a combination of the flow through a
node and the entropy of the node itself. The novel definition of entropy of a node that is provided in this paper is dictated by the routing choices drivers made instead of by purely topological factors.

We have observed that the importance of nodes can vary throughout the day due to changes in traffic patterns, moreover we have designed a method that finds the most robust sensor placement against such type of changes, which we call short term traffic demand variations. Long term changes are also being addressed by our work. We have defined a method to simulate long term city dynamics and their effect on the traffic demand in the city. Moreover, a method is described in order to find a robust sensor placement against such types of changes, providing certainty that sensors will not have to be moved once they are installed.

Finally, we have designed a method that allows designers to weigh various factors connected to their preferences regarding the sensor network and its functionality, in order to determine the optimal number of sensors that need to be placed. The utility function consists of the variation factor of the sensor readings, the average percentage of mismatched sensors under varying traffic demand, the performance and the sensor installment and sustaining cost.

Future work on this topic would require the development of a more comprehensive tool for modelling long term changes in city dynamics, such as building new living or business areas, building new road segments etc. Implementation of such type of changes will bring qualitatively different type of OD matrix variations, since this processes will create completely new origins and destinations. Moreover the population growth should be modelled as well.

Another interesting research aspect would be to use the measure of importance in order to reconstruct drivers’ trajectories. Due to the fact, that, by design, the sensed locations maximise the information about drivers’ routing choices, the sensed data can be very useful in order to determine the paths commuters take and consequentially the OD matrix. However, attention should be given to the issue that regions where commuters actually begin or end their trips are usually of low importance value.

The heterogeneity of the nodes’ importance as a network characteristic can be of great importance as well, as it is directly proportional to the utilization factor of the transportation network. In case of homogeneous importance values, there is lack of central points at which congestion is created. Homogeneity of the importance measure also means that drivers are evenly spread across the network and utilize fully its infrastructure. Heterogeneity, on the other hand, means that drivers’ paths are very similar with the exception of several hub points through which everyone passes. This might bring imbalance of traffic on the network as some roads become congested while others stay empty. Following this argument it might be interesting to use the measure of heterogeneity of the importance measure of a network in order to either evaluate the traffic performance or optimise the routing of commuters leading to overall reduction of congestion.

Finally, an interesting application of the measure would be to use its daily variation to determine the nodes (cross-sections) that are most dynamical in a city. Those intersections should be given additional attention. For example, this measure might be used in order to find the optimal locations for installing intersection control systems, since it provides us with the information that flow ratios at those nodes vary significantly throughout the day.

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REFERENCES


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