Asymptotic Motion Control of Robot Manipulators Using Uncalibrated Visual Feedback*

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Abstract

To implement a visual feedback controller, it is necessary to calibrate the homogeneous transformation matrix between the robot base frame and the vision frame besides the intrinsic parameters of the vision system. The calibration accuracy greatly affects the control performance. In this paper, we address the problem of controlling a robot manipulator using visual feedback without calibrating the transformation matrix. We propose an adaptive algorithm to estimate the unknown matrix on-line. It is proved by Lyapunov approach that the robot motion approaches asymptotically to the desired one and the estimated matrix is bounded under the control of the proposed visual feedback controller. The performance has been confirmed by simulations and experiments.

1 Introduction

Visual feedback is an important approach to improve the control performance of robot manipulators [1]. In order to conduct high precision visual feedback manipulation, some key parameters such as the homogeneous transformation matrix between the robot base frame and the vision frame in position-based methods [2]-[5], or those concerning image Jacobian in image-based methods [6]-[11] should be calibrated accurately besides the intrinsic parameters of the vision system. However, an accurate calibration requires substantial efforts and time, and is also impossible in some cases such as when the vision system is mounted on a mobile platform (robot). For this reason, tremendous efforts have been recently made to visual feedback control with uncalibrated vision system. Papanikolopoulos et al. [7] proposed an algorithm based on on-line estimation for the relative distance of the target with respect to the camera. This algorithm obviates the need for off-line calibration of the eye-in-hand robotic system. Yoshimi et al. [10] utilized a simple geometric property, that is rotational invariance under a special setup of system for a peg-in-hole alignment task, to estimate image Jacobian. Hosada et al. [8] and Jägersand et al. [9] employed the Broyden updating formula to estimate the image Jacobian. The methods in [11] by Kim et al. do not use depth in the feedback formulation. However, the methods above considered kinematics only and neglected dynamic effect of the robot manipulator. To achieve high-lever performance for a manipulator-vision system, the controller must incorporate the dynamics of the manipulator.

In this paper, we address the design of a position-based visual feedback controller for motion control of a robot manipulator when the homogeneous transformation matrix between the robot base frame and the vision frame is not calibrated. It is assumed that the intrinsic parameters of the vision system have been calibrated accurately and the vision system can measure the 3D position and orientation of the end-effector of manipulator. Based on an important observation that the visual Jacobian matrix can be represented as a product of a known matrix, which depends on the kinematics of the manipulator, and the unknown rotation matrix R between the robot base frame and the vision frame, we propose a simple adaptive algorithm to estimate the unknown matrix on-line. The controller can be considered as a combination of an on-line calibration and the real-time control. It is proved with a full consideration of dynamics of the system by Lyapunov approach that this controller yields asymptotic convergence of the motion error to zero and the estimated matrix is bounded. The performance of controller has been verified by simulations and experiments.

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2 Kinematics and Dynamics

2.1 The Coordinate Frames

Fig. 1 shows a typical set-up of a robot workcell using a visual feedback. Three coordinate frames, namely the robot base frame $\sum_B$, the end-effector coordinate frame $\sum_E$, and the vision frame $\sum_V$, are defined, respectively. Here, the $\nu T_B$ is generally represented as:

$$V T_B = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix},$$

where $R \in \mathbb{R}^{3 \times 3}$ denotes the rotation matrix. $p \in \mathbb{R}^3$ is position of the origin of $\sum_B$ with respect to $\sum_V$. Assume that the intrinsic parameters of the vision system have been calibrated, and that the vision system can measure the 3D position and orientation of robot in real-time. However the homogeneous transformation matrix $V T_B$ of the robot base frame with respect to the vision frame is unknown.

![Diagram of coordinate frames](image)

Figure 1: The coordinate frames.

2.2 Kinematics of System

Denote by $^B x_E = [^b x_{e1}, ^b x_{e2}, \ldots, ^b x_{e6}]^T$ the position and orientation of the end-effector with respect to the robot base frame. The first three components of $^B x_E$ denote the position, and the last three are the roll, pitch and yaw angles representing the orientation. Let $x \in \mathbb{R}^6$ denotes the position and orientation of the end-effector with respect to the vision frame. Denote by $q$ the joint angles of the robot. From the forward kinematics, we have

$$^B \dot{x}_E = J(q) \dot{q}$$

where $^B \dot{x}_E \in \mathbb{R}^6$ is the velocity of the end-effector. $J(q)$ is the Jacobian matrix of the robot. $\dot{q}$ denotes the joint velocity. According to the relation of kinematics, we also have

$$\dot{x} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} J(q) \dot{q}$$

where $\dot{x} \in \mathbb{R}^6$ denotes the velocity of the end-effector with respect to the robot base frame. The matrix $A J(q)$ is called visual Jacobian matrix. Assuming that $J(q)$ is square and nonsingular, we then have

$$\dot{q} = J^{-1}(q) \begin{bmatrix} R^T & 0 \\ 0 & R^T \end{bmatrix} \dot{x}$$

Differentiating the equation (3) results in

$$\ddot{q} = J^{-1}(q) A^T \ddot{x} + \frac{d}{dt} (J^{-1}(q) A^T) \dot{x}$$

Note that $R$ in equations (2)~(4) is unknown if no calibration is performed.

2.3 The System Dynamics

The dynamics of robot in the joint space can be represented as

$$H(q) \ddot{q} + \left( \frac{1}{2} H(q) + S(q, \dot{q}) \right) \dot{q} + G(q) = \tau$$

where $H(q)$ is the symmetric and positive definite inertia matrix. $S(q, \dot{q})$ denotes a skew symmetric matrix. $G(q)$ is the gravity force. The $\tau$ represents the joint input of the manipulator.

3 Position Control

In this section, we consider the problem of moving the end-effector of robot from a position $x$ to a desired one $x_d$. Firstly, we adopt the popular PD plus gravity compensation scheme for position control:

$$\tau = G(q) - K_v \dot{q} - K_p J^T(q) \begin{bmatrix} \dot{R}^T & 0 \\ 0 & \dot{R}^T \end{bmatrix} \Delta x$$

where $\dot{R}^T$ denotes an estimated value of transposed matrix of $R$. $\dot{A}^T$ represents the estimated value of transposed matrix of $A$. $\Delta x = x - x_d$ denotes the position error with respect to the vision frame. $K_p$ and $K_v$ are the positive scalar constant gains. Substituting this control law into equation (5) results in the following closed-loop dynamics equation:

$$H(q) \ddot{q} + C(q, \dot{q}) \dot{q} = -K_v \dot{q} - K_p J^T(q) \dot{A}^T \Delta x$$

where $C(q, \dot{q})$ is a function of $q$ and $\dot{q}$.
Note that the right side of equation (7) can be rewritten as follows:
\[
H(q)\ddot{q} + C(q, \dot{q})\dot{q} = -K_v\dot{q} - K_pJ^T(q)AT\Delta x - K_pJ^T(q)(\dot{A}_T - \dot{A}^T)\Delta x
\]
\[
= -K_v\dot{q} - K_pJ^T(q)AT\Delta x - K_pJ^T(q)Y(\Delta x)\Delta \Theta
\]
where we arrange the elements of \( R \) into a \( 9 \times 1 \) vector \( \Theta \). \( \hat{\Theta} \) is an estimated value of \( \Theta \) and \( \Delta \Theta = \hat{\Theta} - \Theta \). The \( Y(\Delta x) \) is a regressor matrix without depending on any element of \( R \) and \( \hat{R} \). Obviously, based on the fact that \( R \) is an orthonormal matrix, we draw into the following two approaches for leading the estimated rotation matrix to tend to the actual rotation matrix. In the first approach, we select a vector \( \kappa \), and then calculate
\[
(\hat{R}^T \hat{R} - I)\kappa = (\hat{R}^T \hat{R} - R^T R)\kappa
\]
\[
= \hat{R}^T(\hat{R} - R)\kappa + (\hat{R}^T - R^T)\frac{RK}{w_1}
\]
\[
= Y_2(\hat{\Theta}, \kappa, w_1)\Delta \Theta
\]  
where \( I \) denotes a \( 3 \times 3 \) identity matrix. \( Y_2(\hat{\Theta}, \kappa, w_1) \) is a \( 3 \times 9 \) regressor matrix which does not depend on the elements of \( \Delta \Theta \). If the vector \( \kappa \) is defined with respect to the robot base frame, \( w_1 \) is a vector with respect to the vision frame. When the vector \( \kappa \) is properly selected that its coordinates with respect to the robot base frame and the vector \( w_1 \) with respect to the vision frame can be measured by the encoders and the vision system respectively, the matrix \( Y_2(\hat{\Theta}, \kappa, w_1) \) can be calculated without using unknown \( R \). In the second approach, we define
\[
\hat{R}\zeta - R\zeta = Y_2(\zeta)\Delta \Theta
\]  
where \( \zeta \) is a vector on the robot. \( w_2 \) is a vector with respect to the vision frame, so \( w_2 \) can be measured by the vision system.

Based on the two approaches in eqs (9) or (10), we propose the following updated laws to calculate the estimated \( \hat{\Theta} \), respectively:
\[
\Delta \hat{\Theta}^T = -\left[ (\hat{R}^T \hat{R} - I)\kappa \right] B_2 Y_2(\hat{\Theta}, \kappa, w_1)
\]
\[
+ \frac{1}{B_1} q^T K_pJ^T(q)Y(\Delta x)
\]
\[
= \frac{1}{B_1} q^T K_pJ^T(q)Y(\Delta x)
\]
or
\[
\Delta \hat{\Theta}^T = -\left[ (\hat{R}^T - R)\kappa \right] B_2 Y_2(\zeta)
\]
\[
+ \frac{1}{B_1} q^T K_pJ^T(q)Y(\Delta x)
\]
where \( B_1 \) and \( B_2 \) are the positive constant gains. From the adaptive laws, we have
\[
q^T K_pJ^T(q)Y(\Delta x)\Delta \Theta = \hat{\Theta}^T B_1 \Delta \Theta + \Delta \Theta^T Y_2(\zeta) B_1 B_2 Y_2(\zeta) \Delta \Theta
\]  
where \( Y_2(\zeta) \) represents either \( Y_2(\hat{\Theta}, \kappa, w_1) \) or \( Y_2(\zeta) \) respectively in two approaches.

**Theorem 1:** Under the control of the proposed controller in equation (6), the robot manipulator system yields
- the estimated matrix \( \hat{R} \) is bounded, and
- asymptotic convergence of position error \( \Delta x \) with respect to the vision frame to zero as the time approaches to the infinity.

A Proof can be referred to [15].

### 4 Trajectory Tracking Control

In this section, we consider the problem of controlling the end-effector of the robot to trace a given time-varying trajectory \((x_d(t), \dot{x}_d(t), \ddot{x}_d(t))\) with respect to the vision frame. Firstly we define the following nominal reference with respect to the vision frame
\[
\hat{x}_r = \hat{x}_d - \lambda \Delta x
\]
\[
\hat{\dot{x}}_r = \hat{x}_d - \lambda \Delta \dot{x}
\]  
where \( \lambda \) is a positive constant. \( \Delta x = x - x_d \) and \( \Delta \dot{x} = \dot{x} - \dot{x}_d \) denote the position and velocity errors with respect to the vision frame, respectively. The error vector is given by
\[
s = \hat{x} - \hat{x}_r = \Delta \dot{x} + \lambda \Delta x
\]

Referring to the nominal reference and the error vector, the joint space nominal reference and the error vector are given as follows:
\[
\hat{q}_r = J^{-1}(q)\dot{A}_T \hat{x}_r
\]
\[
s_q = \dot{q} - \hat{q}_r = \dot{q} - J^{-1}(q)\dot{A}_T \hat{x}_r
\]

From equation (3), we can also re-write \( s_q \) as:
\[
s_q = J^{-1}(q)(A^T - \hat{A}^T)\dot{x} + J^{-1}(q)\dot{A}^T \dot{x}
\]

Using the computed-torque method, we propose the following control law for trajectory tracking:
\[
\tau = -K_pJ^T(q)\dot{A}_T \dot{x} + K_v s_q + G(q)
\]
\[
+ H(q)(J^{-1}(q)\dot{A}_T \dot{x}_r + (J^{-1}(q)\dot{A}_T)\dot{x}_r) + C(q, \dot{q})\dot{q}_r
\]  
where \( B_1 \) and \( B_2 \) are the positive constant gains. From the adaptive laws, we have
where $K_p$ and $K_v$ are the positive scalar constant gains. Substituting the control law into equation (5) results in the following closed-loop dynamics equation:

$$H(q)s_q + C(q,q)s_q = -K_p J^T(q) \dot{A}^T s - K_v s_q$$  \hspace{1cm} (20)

An adaptive law is necessary to update the estimated matrix $\hat{A}$. Consider the following equation

$$\hat{A}^T \dot{x} - \hat{A}^T \dot{\hat{x}} = Y_3(\dot{x})\Delta \Theta$$  \hspace{1cm} (21)

where $Y_3(\dot{x})$ is a regressor matrix without depending on elements of $R$. The vector $v_f$ is the velocity of the end-effector with respect to the robot base frame and can be measured by the encoders. Furthermore,

$$\dot{x}^T (\hat{A} - A) K_p \hat{A}^T s = Y_4(\dot{x}, K_p, \hat{A}, s) \Delta \Theta$$  \hspace{1cm} (22)

Note that the regressor matrix $Y_4(\dot{x}, K_p, \hat{A}, s)$ does not depend on the elements of $\Delta \Theta$.

Then, the following adaptive law is proposed:

$$\Delta \dot{\Theta}^T = - \left( \frac{\dot{\hat{A}}^T \dot{\hat{x}} - v_f^T}{\Delta \Theta^T Y_4^T(\dot{x})} \right) B_3 \dot{Y}_3(\dot{x}) - \frac{1}{B_4} Y_4(\dot{x}, K_p, \hat{A}, s)$$  \hspace{1cm} (23)

where $B_3$ and $B_4$ are the positive constant gains. From this equation, we have

$$\Delta \dot{\Theta}^T B_4 \Delta \Theta = - \Delta \Theta^T Y_4 B_4 \dot{B}_4 B_3 \dot{Y}_3 \Delta \Theta - Y_4 \dot{\Delta} \Theta$$  \hspace{1cm} (24)

**Theorem 2:** Under the control of the proposed controller in equation (19), the robot manipulator system yields

- the error $(\hat{R} - R)$ is bounded, and
- asymptotic convergence of trajectory tracking errors $\Delta x(t)$ and $\Delta \dot{x}(t)$ with respect to the vision frame to zero as the time approaches to the infinity.

**Proof:** Define the following nonnegative scalar function

$$V = \frac{1}{2} \left\{ s_q^T H(q)s_q + \Delta \Theta^T B_4 \Delta \Theta \right\}$$  \hspace{1cm} (25)

By multiplying the closed-loop dynamics equation (20) from the left-hand side by $s_q^T$ derives

$$s_q^T H(q)s_q + s_q^T \frac{1}{2} \dot{H}(q)s_q = -s_q^T K_p J^T \hat{A}^T s - s_q^T K_v s_q$$  \hspace{1cm} (26)

From equation (18), note that the term in the equation (26)

$$-s_q^T K_p J^T \hat{A}^T s = \dot{x}^T (\hat{A} - A) K_p \hat{A}^T s - (\hat{A}^T s)^T K_p (\hat{A}^T s)$$  \hspace{1cm} (27)

Differentiating the nonnegative scalar function in equation (25) and then substituting eqs (24), (26) and (27) into it, we obtain

$$\dot{V} = -\Delta \Theta^T Y_3^T B_3 B_4 \dot{Y}_3 \Delta \Theta - s_q^T K_v s_q - (\hat{A}^T s)^T K_p (\hat{A}^T s)$$  \hspace{1cm} (28)

For the positive constants $K_v, K_p, B_3$ and $B_4$, $\dot{V}$ is nonpositive and hence $V$ never increases. It states that $V$ is a Lyapunov function. From Barbalat’s lemma, we can say the error $R - \hat{R}$ is bounded. Also, we have asymptotic convergence of the trajectory tracking errors $\Delta x$ and its successive derivative $\Delta \dot{x}$ with respect to the vision frame to zero. □

5 Simulations

In this section, we show the performance of the proposed trajectory tracking controller by simulations. We conducted the simulations on a two-link planar arm with the physical parameters $m_1 = m_2 = 1, l_1 = l_2 = 2$, as shown Fig. 1. The arm base frame is located at $(-10, -10)$ with respect to the vision frame. In the simulations, the rotation matrix $R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ is a function of $\theta$. The end-effector of the arm is required to follow the following desired trajectory

$$x_d(t) = \begin{bmatrix} -0.8 \sin(\omega t) - 9.0 \\ 0.8 \cos(\omega t) - 8.5 \end{bmatrix}$$

with respect to the vision frame. All units in the simulations used are in the SI system. The simulation results are plotted in Fig. 2. In the simulations, the real rotation matrix $R(\theta = \pi/2)$, the initial estimation of the rotation matrix $\hat{R}(\theta = \pi/5)$; the positive gains are $K_p = 40, K_v = 30, B_3 = 30, B_4 = 200; \lambda = 3, \omega = 1$; the initial position of the end-effector is $x_0 = (-8.5, -9.0)$. As shown in Fig. 2, the results confirmed asymptotic convergence of the tracking errors and the bounded $R$.

6 Experiments

We have implemented the controller in the five-fingered robot hand system developed at the Chinese University of Hong Kong using DSP’s and workstations. Each finger of the robot hand has three revolute joints driven by AC motors through a harmonic drive of 80:1 reduction ratio. The joint angles of the
resolutions of 30720 pulse/turn. The joint velocities are obtained by differentiating the joint angles. In the experiments, one finger of the robot was employed as a 3DOF arm. About 2m away from this 3DOF finger, we set an OPTOTRAK/3020 position sensor system to measure in real-time the 3D positions of markers mounted at the fingertip with resolution of 1:200000 (power axis). Like the joint velocities, we can also obtain the velocities of the fingertip by differentiating the positions detected by OPTOTRAK sensors. In order to communicate the data, an interface board is installed in a PC with Intel80486 CPU between the robot hand and OPTOTRAK system. To compensate for the frictions at the joints, we adopt the following friction model:

$$ F = K_{ef} \dot{q} + [K_{df} + (K_{sf} - K_{df})dexp]sgn(\dot{q}) $$

where $dexp = diag\{exp(-|q_i|/\alpha)\}$. $K_{ef} = diag\{0.1, 0.1, 0.1\}$ is the coefficient matrix of viscous friction, $K_{sf} = diag\{0.6, 0.4, 0.5\}$ is the coefficient matrix of static friction and $K_{df} = diag\{0.4, 0.2, 0.35\}$ is the coefficient matrix of dynamic friction. $\alpha = 0.001$ is a small positive parameter. The sampling time of the experimental system is 2.36 ms. Note that all units in the experiments are in the SI system. Fig. 3 shows the robot manipulator and the vision system.

6.1 Position Control

Two experiments have been conducted to validate the proposed position control scheme using the different approaches in the equation (9) and the equation (10), respectively. We set two sensor markers of OPTOTRAK as a vector whose distance is 0.039m at the robot fingertip. The initial estimation of the rotation matrix $R$ between the finger base frame and the vision frame is given as a $3 \times 3$ identity matrix. The gains are chosen as: $K_p = 250$, $K_v = 10$, $B_1 = 600$, $B_2 = 100$. Fig. 4(a) and (b) show the experimental results of two approaches, respectively. As shown in Fig. 4, the results ascertain the effectiveness of the two proposed control algorithms.

6.2 Trajectory tracking Control

In this experiment, the end-effector of the robot is required to trace a given trajectory

$$ x_d(t) = \begin{bmatrix}
0.006 \cos(\omega t) + 0.028 \sin(\omega t) - 0.163 \\
-0.027 \cos(\omega t) + 0.002 \sin(\omega t) - 0.257 \\
0.012 \cos(\omega t) - 0.01 \sin(\omega t) - 1.802
\end{bmatrix} $$

with respect to the vision frame. We set a marker to detect the change of position of the end-effector. The initial estimation of the rotation matrix is also set as the identity matrix. The initial position is $x_0 = (-0.140071, -0.237624, -1.792197)$; the parameters are set to: $K_p = 50$, $K_v = 30$, $B_3 = 100$, $B_4 = 500$, $\lambda = 3$, $\omega = 1$. The results in Fig. 5 confirmed good convergence of the trajectory tracking errors.
7 Conclusions

In this paper, we proposed a motion controller using visual feedback without calibrating the homogeneous transformation matrix between the robot base frame and the vision frame before executing task. Differing from other approaches, the controller is developed by considering a full dynamics of the system and using an adaptive algorithm to estimate the unknown matrix on-line. The proposed adaptive algorithm is based on an important observation, that is the visual Jacobian matrix can be represented as a product of a known matrix, which depends on the kinematics of the manipulator, and the unknown rotation matrix $R$ between the robot base frame and the vision frame. This controller greatly simplifies the implementation process of a robot-vision workcell and is especially useful when a pre-calibration is impossible. Simulation and experimental results verified the performance of asymptotic convergence of the new controller. The future work is to extend this method to the image-based visual feedback control.

References


