ADAPTIVE VISUAL SERVOING FOR CONSTRAINED ROBOTS UNDER JACOBIAN, JOINT DYNAMIC AND CONTACT VISCOSOUS FRICTION UNCERTAINTIES

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Abstract: Visual servoing of constrained robots has not yet met a formal treat-
ment nor its friction compensation. This kind of robots moves slowly along the
constrained surface due to technological limitations of the camera system, therefore
important problems of friction at the joint and contact point arise. The problem
turns very complicated when parametric uncertainty on robot, camera and friction
is considered. In this paper, a new visual servoing scheme that satisfy this problem
is presented. It induces sliding modes without chattering to guarantee locally
exponential convergence of tracking errors. Simulations results are presented and
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Keywords: Visual Servoing, Adaptive Control, Force Control, Uncertain
Jacobian, Friction.

1. INTRODUCTION

It is well known that multisensor-based robot
control approaches may offer a solution to very
important and relevant, but complex, problems
in robotics. In order to achieve this sensor fusion-
based controller, a careful analysis of the dynami-
cs, sensors behavior, and tasks are required. One
of such tasks, is the image-based force-position
control of robots. Furthermore, since parameters
are in practice uncertain, the control must be
robust to all parameters of the system. This sort
of control laws are able to interact under changing
environment and deal with uncertainties of its
dynamic models without the explicit intervention
of humans. Concretely, the desired task is that the
robot end effector tracks a visual trajectory along
the surface of an object, and at the same time,
control the applied force, see Figure 1.

Figure 1. Robot-Force-Vision System.
On the other hand, joint friction and contact friction are quite important to compensate in any practical application. Therefore, we consider for the joint friction the LuGre model, which reproduces a typical motion regime of image-based robotic tasks, and viscous friction at the contact point.

1.1 Background

Good sensing abilities of relevant variables are essential to gain higher flexibility and autonomy of robots in an unknown working environment. Often, either vision or force sensor are used. Hybrid vision/force control approaches have been reported e.g. (J. Baeten, 2000), (Xiao, 2000) and none of them shows robustness to uncertainties, neither robot parameters nor camera parameters.

With respect to force control, Arimoto (Arimoto, 1996) solved by first time the simultaneous control of position and force using the full nonlinear dynamics subject to parametric uncertainties without coordinate partitioning, based on the orthogonalization principle. Afterwards, several schemes have been proposed based on the orthogonalization principle, however these schemes have not been extended or combined with visual servoing beyond free motion robots.

In this paper, an adaptive controller driven by image and contact force errors to solve the problem posed above is proposed. The system guarantees exponential tracking of position and force trajectories subject to parametric uncertainties. This scheme delivers a smooth controller and presents formal stability proofs. Simulations allow to visualize the expected closed loop performance predicted by the theory.

2. NONLINEAR ROBOT DYNAMICS

The constrained robot dynamics arises when its end effector is in contact with infinitely rigid surface. Considering the generalized position \( q \in \mathbb{R}^n \) and velocity coordinates \( \dot{q} \in \mathbb{R}^n \), this system is (Arimoto, 1996)

\[
H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + J_T(q)\lambda - F(q, \psi)
\]

where \( B_t \in \mathbb{R}^{n \times n} \) stands for the viscous friction matrix and possibly not a diagonal matrix, \( J_T(q)B_tJ(q)\dot{q} \) represents the tangential viscous friction in the contact point, \( H(q) \in \mathbb{R}^{n \times n} \) stands for the inertia matrix; \( C(q, \dot{q})\dot{q} \in \mathbb{R}^n \) stands for the vector of centrifugal and Coriolis torques; \( g(q) \in \mathbb{R}^n \) is the vector of gravitational torques, \( F(\dot{q}, \psi) \) is the joint dynamic friction \( ^4 \), \( J_\varphi(q) \) is constrained jacobian of the rigid and frictionless constraint surface \( \varphi(q) = 0 \), and \( \lambda \) is the constrained Lagrangian or contact force.

Adding and subtracting to (1) the linear parameterization \( H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + J_T(q)B_tJ(q)\dot{q} = Y_t\theta_t \), where the known regressor \( Y_t = Y_t(q, \dot{q}, \ddot{q}, \gamma) \in \mathbb{R}^{n \times p} \) and the unknown constant vector \( \theta_t \in \mathbb{R}^p \), produces the open loop error equation

\[
H(q)\dot{S}_q = -C(q, \dot{q})S_q - J_T(q)B_tJ(q)\dot{S}_q + \tau + J_T(q)\lambda - Y_t\theta_t
\]

with joint error surface \( S_q \) is defined as

\[
S_q = \dot{q} - \dot{q}_r
\]

where \( \dot{q}_r \) stands for the nominal reference.

3. CAMERA MODEL

The static pin hole with thin lens without aberration camera model is used (S. Hutchinson, 1996). To introduce it, consider the forward kinematics \( x_b = f(q) \), for \( x_b \in \mathbb{R}^n \) represents the position of robot end effector in cartesian space. Differential kinematics relates velocities in cartesian space to joint space velocities as follows \( \dot{x}_b = J(q)\dot{q} \). According to (S. Hutchinson, 1996), visual position \( x_s \in \mathbb{R}^2 \) of robot end effector in image space is given by \( x_s = \alpha R(\theta)x_b + \beta \) where \( \alpha \) is the scale factor \( ^5 \), \( R(\theta) \in SO(3) \), \( \beta \in \mathbb{R}^2 \) that depends on intrinsic and extrinsic camera parameters. The differential kinematics of camera model is then \( \dot{x}_s = \alpha R(\theta)\dot{x}_b \). Using these relationships, we have an equation that relates \( \dot{x}_s \in \mathbb{R}^2 \) and \( \gamma \in \mathbb{R}^2 \) as follows:

\[
\dot{x}_s = \alpha R(\theta)J(q)\dot{q} \iff \dot{q} = J_{Rinv}\dot{x}_s
\]

whit \( J_{Rinv} = J(q)^{-1}R(\theta)^{-1} \alpha^{-1} \). This relation is useful to design the nominal reference.

4. VISUAL-FORCE EXTENDED ERROR

Since \( \varphi(q) = 0 \forall t \), then its time derivative yields \( \dot{\varphi} = \frac{\partial \varphi(q)}{\partial q} \dot{q} = J_\varphi(q)\dot{q} = 0 \), this means that \( J_\varphi(q) \) is orthogonal to \( \dot{q} \). That is, \( \dot{q} \) belongs to the orthogonal projection matrix \( Q \) of \( J_\varphi(q) \) (Arimoto, 1996)

\[
Q = I - \frac{J_\varphi(q)}{||J_\varphi(q)||^2}J_\varphi(q)^T
\]

As we can see, \( Q \) spans the tangent plane at the contact point, therefore, \( J_\varphi \) and \( Q \) are orthogonal complements. In other words, \( Q\dot{q} = \dot{q} \rightarrow QQ\ddot{q} = \)

\footnote{For a clear exposition, firstly, \( F(\dot{q}, \psi) \) will be considered zero, however in Section 7 it will be treated.}

\footnote{Without loss of generality, \( \alpha \) can be considered as a scalar matrix \( 2 \times 2 \).}
Furthermore, let the nominal visual reference be
\[
\dot{x}_r = \dot{x}_{sd} - \alpha \Delta s_x + S_{sd} - \gamma \int_{t_0}^{t} \text{sign}(S_{sd})
\] (8)
where \(\dot{x}_{sd}\) stands for desired visual velocity trajectory and \(\Delta s_x = x_x - x_{sd}\) is the visual position error, and \(S_x = \Delta \dot{x}_x + \alpha \Delta s_x\), \(S_{sd} = S_{sd}(t_0)\). 

Using equations (7) into (4), we obtain the following orthogonalized joint error surface \(\dot{S}_q = \dot{q} - \dot{q}_r \equiv Q\dot{J}_{Rinv}\ddot{x}_r + \beta J_{p}^T\dot{q}_{rf}\) (9)

with \(J_{Rinv} = J_{Rinv}(q)\), such as rank \(J^{-1}(q)\) and full rank \(\forall q \in \Omega\), where the robot workspace free of singularities is defined by \(\Omega = \{q | \text{rank}(J(q)) = n\}\), and \(\forall \theta \in \mathbb{R}\). Thus, substituting (11) into (4) we have the uncalibrated joint error surface
\[
\dot{S}_q = \dot{q} - \dot{q}_r = Q\dot{J}_{Rinv}\ddot{x}_r - Q\dot{J}_{Rinv}\dot{x}_r - \beta J_{p}^T\dot{q}_{rf}
\] (12)
where \(\dot{S}_q\) is available because \(\dot{q}\) and \(\dot{q}_r\) are available.

Using (11), the uncertain parametrization is
\[
H(q)\ddot{q}_r + C(q, q)\dot{q}_r + g(q) + J_{p}^T(q)B_{r}\dot{J}(q)\dot{q}_r = Y_{cont}\dot{\theta}_b
\]
where \(\dot{q}_r = f(x_r, \dot{x}_r, \dot{q}_r)\), and \(\dot{x}_r = \dot{x}_{sd} - \alpha \Delta s_x + \dot{S}_{sd} - \gamma \text{sign}(S_{sd})\). \(\dot{q}_{rf} = \Delta \dot{F} - \dot{S}_{df} + \gamma F\text{sign}(S_{F3})\), which produces a discontinuous \(\ddot{q}_r\).

To avoid introducing high frequency discontinuous signals, add and subtract \(z_s = \text{tanh}(\nu_s S_{sd})\) and \(z_f = \text{tanh}(\nu F S_{F3})\) to \(\ddot{q}_r\) to separate continuous and discontinuous signals as follows
\[
\ddot{q}_r = \ddot{q}_{cont} + Q\ddot{S}_z - \beta J_{p}^T y_{z_f}
\] (13)
with \(z_s = \text{tanh}(\lambda_s S_{sd}) - \text{sign}(S_{sd})\) and \(z_f = \text{tanh}(\lambda F S_{F3}) - \text{sign}(S_{F3})\). This results that \(Y_{cont} = Y_{r}(q, \dot{q}, \dot{q}_r, \dot{q}_{cont})\) is continuous since \((\dot{q}_r, \dot{q}_{cont}) \in C^1\), where
\[
\dot{\ddot{q}}_{cont} = Q\dot{J}_{Rinv}\ddot{x}_r + \beta J_{p}^T \ddot{q}_{cont} + Q\dot{J}_{Rinv}\dot{x}_r + \beta J_{p}^T \dot{q}_{rf}
\] (14)

Remark 1. The above definition assumes exact knowledge of \(J_{Rinv}\), however, in practice it is a very restricted assumption. Therefore, we need to design uncertain manifold \(S_q\) taking into consideration uncertainty of \(J_q, R(\theta), \alpha\).

To this end, consider
\[
\ddot{S}_q = \dot{q} - \dot{q}_r = Q\dot{J}_{Rinv}\ddot{x}_r - Q\dot{J}_{Rinv}\dot{x}_r - \beta J_{p}^T\dot{q}_{rf}
\] (11)
with \(\dot{J}_{Rinv}\) an estimated \(J_{Rinv}\), such as rank \(J^{-1}(q)\) and \(R_n^{-1}(\theta)\) are full rank \(\forall q \in \Omega\), where the robot workspace free of singularities is defined by \(\Omega = \{q | \text{rank}(J(q)) = n\}\), and \(\forall \theta \in \mathbb{R}\). Thus, substituting (11) into (4) we have the uncalibrated joint error surface
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\] (12)
where \(\dot{S}_q\) is available because \(\dot{q}\) and \(\dot{q}_r\) are available.

5. OPEN LOOP ERROR EQUATION

Using (11), the uncertain parametrization is
\[
H(q)\ddot{q}_r + C(q, q)\dot{q}_r + g(q) + J_{p}^T(q)B_{r}\dot{J}(q)\dot{q}_r = Y_{cont}\dot{\theta}_b
\]
where \(\dot{q}_r = f(x_r, \dot{x}_r, \dot{q}_r)\), and \(\dot{x}_r = \dot{x}_{sd} - \alpha \Delta s_x + \dot{S}_{sd} - \gamma \text{sign}(S_{sd})\). \(\dot{q}_{rf} = \Delta \dot{F} - \dot{S}_{df} + \gamma F\text{sign}(S_{F3})\), which produces a discontinuous \(\ddot{q}_r\).

To avoid introducing high frequency discontinuous signals, add and subtract \(z_s = \text{tanh}(\nu_s S_{sd})\) and \(z_f = \text{tanh}(\nu F S_{F3})\) to \(\ddot{q}_r\) to separate continuous and discontinuous signals as follows
\[
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\[
\dot{\ddot{q}}_{cont} = Q\dot{J}_{Rinv}\ddot{x}_r + \beta J_{p}^T \ddot{q}_{cont} + Q\dot{J}_{Rinv}\dot{x}_r + \beta J_{p}^T \dot{q}_{rf}
\] (14)

Remark 1. The above definition assumes exact knowledge of \(J_{Rinv}\), however, in practice it is a very restricted assumption. Therefore, we need to design uncertain manifold \(S_q\) taking into consideration uncertainty of \(J_q, R(\theta), \alpha\).

To this end, consider
\[
\ddot{S}_q = \dot{q} - \dot{q}_r = Q\dot{J}_{Rinv}\ddot{x}_r - Q\dot{J}_{Rinv}\dot{x}_r - \beta J_{p}^T\dot{q}_{rf}
\] (11)
with \(\dot{J}_{Rinv}\) an estimated \(J_{Rinv}\), such as rank \(J^{-1}(q)\) and \(R_n^{-1}(\theta)\) are full rank \(\forall q \in \Omega\), where the robot workspace free of singularities is defined by \(\Omega = \{q | \text{rank}(J(q)) = n\}\), and \(\forall \theta \in \mathbb{R}\). Thus, substituting (11) into (4) we have the uncalibrated joint error surface
\[
\dot{S}_q = \dot{q} - \dot{q}_r = Q\dot{J}_{Rinv}\ddot{x}_r - Q\dot{J}_{Rinv}\dot{x}_r - \beta J_{p}^T\dot{q}_{rf}
\] (12)
where \(\dot{S}_q\) is available because \(\dot{q}\) and \(\dot{q}_r\) are available.

6. CONTROL DESIGN

Theorem 1: Assume that initial conditions and desired trajectories belong to \(\Omega\). Consider the constrained robot dynamics (1), subject to parametric uncertainties, robot and tangential contact friction, in closed loop with the following visual adaptive force-position control law
\[
\tau = -K_4\dot{S}_q + \gamma \dot{q}_{cont}\dot{\theta}_b + \beta J_{p}^T(q) [\alpha S_{F3}] + \gamma F J_{p}^T(q) \text{tanh}(\mu S_{F3}) + \mu S_{F3} \text{sign}(S_{F3})
\] (17)
where \(\Gamma \in R_+^{nxn}, K_4 \in R_+^{nxn}, \mu > 0, \lambda_d \text{ the desired contact force. If } K_4 \text{ is large enough and an}

error of initial conditions are small enough, with $\gamma_s \geq \frac{d}{\pi} \big\{ R_0(\theta)(J(q) \dot{S}_q + (J_{\text{Rinv}} - J_{\text{Rinv}}) \dot{x}_r) \big\}$ and $\gamma_F \geq \frac{d}{\pi} \left( (J_{\varphi} J_{\varphi}^T(q))^{-1} J_{\varphi} \dot{S}_q \right)$ then exponential convergence of visual and force tracking errors is guaranteed.

**Proof:** We prove that the closed loop dynamics (CLD) between (17)-(18) and (1), shows boundedness of all system trajectories, with exponential convergence of tracking errors. The proof is organized in three parts.

### Part I. Boundedness of Closed Loop Trajectories

Consider the time derivative of the following Lyapunov candidate function

$$V = \frac{1}{2} \left[ S_q^T H(q) \dot{S}_q + \beta S_q^T S_q + \Delta \theta^T \Gamma \Delta \theta \right],$$

with $\Delta \theta_0 = \theta_0 - \hat{\theta}$, along the solutions of closed loop dynamics (CLD) as

$$\dot{V} \leq -K_2 \left\| J_{\varphi} \right\|^2_2 - \eta \beta \| S_{qF} \| + \left\| S_q \right\| \psi$$

where $\psi$ is a functional depending on the state and error manifolds. Now if $K_2, \eta$ and $\beta$ are large enough and the initial errors are small enough, we conclude the seminfinite definiteness of $V$ outside of hyperball $\varepsilon_0 = \left\{ \hat{S}_q | V \geq 0 \right\}$ centered at the origin, such as the following properties of the state of closed loop system arise

$$\hat{S}_q, S_{qF} \in L_\infty \Leftrightarrow \left\| S_{qF} \right\| \in L_\infty \tag{19}$$

Then, $(S_{\delta}, \int \text{sign}(S_{\delta})) \in L_\infty$ and since desired trajectories are $C^2$ and feedback gains are bounded, we have that $(\hat{q}_r, \hat{q}_F) \in L_\infty$. The right hand side of (16) shows that $\varepsilon_1 > 0$ exists such that

$$\left\| \hat{S}_q \right\| \leq \varepsilon_1$$

This result shows only local stability of $\hat{S}_q$ and $\hat{S}_q$, now we prove that the sliding modes arises. Adding and subtracting $QJ_{\text{Rinv}} \dot{x}_r$ to (12) we obtain

$$\dot{S}_q = Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \} - \beta J_{\varphi}^T \{ S_{qF} \} \tag{20}$$

where $\Delta J_{\text{Rinv}} = J_{\text{Rinv}} - J_{\text{Rinv}}$. Since $\dot{S}_q \in L_2$, and $J_{\text{Rinv}}$ and $Q$ are bounded, then $QJ_{\text{Rinv}} S_{qF}$ is bounded and, due to $\varphi(q)$ is smooth and lies in the reachable robot space and $S_{qF} \rightarrow 0$, then $\beta J_{\varphi}^T S_{qF} \rightarrow 0$. Now, taking into account that $\dot{S}_q$ is bounded, then $\frac{d}{dt} J_{\text{Rinv}} Q S_{qF}$ and $\frac{d}{dt} \beta J_{\varphi}^T S_{qF}$ are bounded (this is possible because $J_{\varphi}^T$ is bounded and so $\dot{Q}$ is). All this chains of conclusions proves that there exists constants $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ such that $\left\| \hat{S}_{qF} \right\| < \varepsilon_2$, $\left\| S_{qF} \right\| < \varepsilon_3$. Now, we have to prove that for a proper selection of feedback gains $\gamma_s$ and $\gamma_F$, we can conclude that trajectories of visual position and force converges to zero. This is possible if we can prove that sliding modes are established for the subspace of visual position $Q$ and the subspace of force $J_{\varphi}^T(q)$. Considering that operator $QJ_{\text{Rinv}}$ spans the vector $\hat{S}_q$ in its image $\text{im} \{ QJ_{\text{Rinv}} S_{qF} \}$ $\equiv S_{qF}^\text{im}$ and the operator $\beta J_{\varphi}^T$ spans the same vector in its image $\text{im} \{ \beta J_{\varphi}^T S_{qF} \} \equiv S_{qF}^\text{im}$, this implies that

$$\dot{S}_q = Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \} - \beta J_{\varphi}^T \{ S_{qF} \} \equiv \{ S_{qF} \} \tag{21}$$

where $S_{qF}^\text{im} \in \text{im} \{ \Delta J_{\text{Rinv}} \dot{x}_r \}$ and $S_{qF}^\text{im}$ belongs to a orthogonal complements, $\langle S_{qF}^\text{im}, \text{im} \{ \Delta J_{\text{Rinv}} \dot{x}_r \} \rangle = 0$. That is, we are able to analyze the $S_{qF}^\text{im} \in \text{im} \{ \Delta J_{\text{Rinv}} \dot{x}_r \}$ dynamics, independently of $S_{qF}^\text{im}$, since $S_{qF}^\text{im}$ belongs to the kernel of $Q$. This is verified if we multiply (21) for $Q^T$,

$$Q^T \dot{S}_q = Q^T Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \} - \beta Q^T J_{\varphi}^T \{ S_{qF} \} \equiv \{ S_{qF} \} \tag{22}$$

since $Q$ is idempotent ($Q^T Q = Q$). It is important to notice that if $Ax = Ay$ for any square nonsingular matrix $A$ and any couple of vectors $x, y$, then $x \equiv y$. Thus, the equation (22) means that for the subspace $Q$, the equality $\dot{S}_q = Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \}$ is valid within span of $Q$. Now, if we multiply $\hat{S}_q$ for $J_{\varphi}$ to obtain

$$J_{\varphi} \dot{S}_q = J_{\varphi} Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \} - \beta Q^T J_{\varphi}^T \{ S_{qF} \} \equiv \{ S_{qF} \} \tag{23}$$

### Part II: Second Order Sliding Modes

#### Part II.a: Sliding modes for the velocity subspace $Q$

According to $Q^T \dot{S}_q = Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \}$, then $\dot{S}_q \equiv J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r$ in the subspace image of $Q$, however notice that $Q$ is not full rank, then this equality is valid locally, not globally. In this local neighborhood, if we multiply the equality $\dot{S}_q = Q \{ J_{\text{Rinv}} S_{qF} - \Delta J_{\text{Rinv}} \dot{x}_r \}$ by $R_{s\theta}(\theta) J(q) \gamma_s$ 7, we have

$$R_{s\theta}(\theta) J(q) \dot{S}_q = S_{\delta} \dot{s}_\delta + \gamma_s \int \text{sign}(S_{\delta}) - R_{s\theta}(\theta) J(q) \{ \Delta J_{\text{Rinv}} \dot{x}_r \} \tag{24}$$

Taking the time derivative of the above equation, and multiply it by $S_{\delta s}^T$ produces

$$S_{\delta s}^T S_{\delta s} = -\gamma_s S_{\delta s}^T \text{sign}(S_{\delta s}) + S_{\delta s}^T \frac{d}{dt} R_{s\theta}(\theta) J(q) \left( \dot{S}_q + \Delta J_{\text{Rinv}} \dot{x}_r \right) \leq -\mu_s \| S_{\delta s} \|$$

(25)

where $\mu_s = \gamma_s - \varepsilon_4$, and $\varepsilon_4 = \frac{d}{dt} \| R_{s\theta}(\theta) J(q) (\dot{S}_q + \Delta J_{\text{Rinv}} \dot{x}_r) \|$. Thus, we obtain the sliding condition if $\gamma_s > \varepsilon_4$, such as, $\mu_s > 0$ of (25) guarantee the sliding mode at $S_{\delta s} = 0$ in a time $t_s = \| S_{\delta s} \|$. However, notice that for any initial condition $S_{\delta s}(t_0) = 0$, then $t_s = 0$, which implies that the

7 Remember the equality: $J_{\text{Rinv}} = J^{-1}(q) R^{-1}(\theta) \alpha^{-1}$. 
sliding mode at \( S_{\delta} (t) = 0 \) is guaranteed for all time.

**Part II.b: Sliding modes for the force subspace.** Similarly, if we multiply (23) by \((J_{\varphi}^T J_{\varphi})^{-1}\), we obtain

\[
(J_{\varphi}^T J_{\varphi})^{-1} J_{\varphi} \dot{S}_{\delta} = -\beta J_{\varphi}^T \{ S_{\delta F} \}
\]

(26)

\[
J_{\varphi}^T (\dot{q}) S_{\delta} = S_{F \delta} + \gamma_F \int \text{sign}(S_{F \delta})
\]

(27)

where \( J_{\varphi}^T (\dot{q}) = (J_{\varphi}^T J_{\varphi})^{-1} J_{\varphi} \). Derivating the above equation and multiply for \( S_{F \delta} \) lies

\[
S_{F \delta} \dot{S}_{F \delta} = -\gamma_F |S_{F \delta}| + S_{F \delta} \frac{d}{dt} \left( J_{\varphi}^T (\dot{q}) \dot{S}_{\delta} \right)
\]

(28)

\[
\leq -\mu_F |S_{F \delta}|
\]

(29)

where \( \mu_F = \gamma_F - \varepsilon_5 \), and \( \varepsilon_5 = \frac{4}{\pi} \left( (J_{\varphi}^T J_{\varphi})^{-1} J_{\varphi} \dot{S}_{\delta} \right) \). If \( \gamma_F > \varepsilon_5 \), then a sliding mode at \( S_{F \delta} (t) = 0 \) is induced in a time \( t_f \leq \frac{|S_{F \delta}(t_0)|}{\mu_F} \), but \( S_{F \delta}(t_0) = 0 \).

**Part III.a: Exponential convergence of tracking errors.**

**Part III.b: Force tracking errors.** Since a sliding mode exists for all time at \( S_{\delta} (t) = 0 \), then we have that \( S_{\delta} = S_{\delta} d t \rightarrow \Delta \dot{x}_a = -\alpha \Delta x_a + \dot{S}_{\delta}(t_0) \ e^{-\kappa t} \). This implies that the visual tracking errors locally tends to zero exponentially fast, this is \( x_{a} \rightarrow x_{ad} \), \( \dot{x}_{a} \rightarrow \dot{x}_{ad} \), implying that the robot end-effector converges to the desired image \( x_{ad} \), with given velocity \( \dot{x}_{ad} \).

**Remark 2.** We have proved that \( J(q(t)) \) is not singular for all time, because \( q(t) \rightarrow q_{d}(t) \) exponentially, without overshoot, with desired trajectories belonging to robot workspace \( \Omega \), thus \( J(q(t)) \rightarrow J(q_{d}(t)) \) within \( \Omega \).

**Remark 3.** Using continuous \( \tanh(*) \) instead of \( \text{sign(*)} \) induces larger upper bounds \( \varepsilon_2 \) and \( \varepsilon_3 \) in comparison of using \( \text{sign(*)} \), with the great advantage of getting ride of the harmful chattering. In this case, to induce the second order sliding mode, and therefore exponential convergence of tracking errors, it suffices to tune \( \gamma_F \) and \( \gamma_T \) to a larger value.

7. DYNAMIC FRICITION COMPENSATION

Now let us consider the dynamic friction into the model, which represent a very realistic behaviour when the robot is moving along a rigid surface, in particular, driven by visual servoing. In this case, the following LuGre (Canudas de Wit and Astrom, 1995) dynamic friction model is very appropriate to define the joint friction

\[
J(q, \dot{q}) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{q} + \dot{z} = \sigma_0 h(\dot{q}) z + \dot{q} \left( \frac{\dot{q}}{\alpha_0 + \alpha_1 \exp(-\dot{q}/\sigma^2)} \right)
\]

(30)

where matrix parameters \( \sigma_1, \sigma_2, \sigma_3 \) are diagonal definite matrices \( n \times n \), the state \( z \in \mathbb{R}^n \) stands for the position of the bristles, \( \alpha_0, \alpha_1 > 0 \), and \( \sigma_3 > 0 \). This model involve a very complex dynamics around the trivial equilibrium, for bidirectional motion, and for very small displacements, the forces that comes out from this model makes impossible to reach the origin due to limit cycles induced and the potentially unstable behavior. Substituting (30) into (1) yields

\[
H(q, \dot{q}) + J(q, \dot{q}) + \beta J(q) \dot{q} + \sigma_2 \dot{q} + \dot{q} = \gamma F \Theta \dot{f}_w, \text{where } \gamma F > 0. \]

where \( \alpha_0 = \alpha_0 + \alpha_1 \), \( tanh(z) \) is the continuous hyperbolic tangent function, and \( \lambda_f > 0 \). Now, if we add and subtract the above regressors to (31) yields the following open-loop error dynamics with dynamic friction

\[
H(q, \dot{q}) = -\left( (C(q, \dot{q}) + \beta J(q) \dot{q} + \sigma_2 \dot{q} + \dot{q} = \gamma F \Theta \dot{f}_w, \text{where } \gamma F > 0. \]

\[
Y = [ Y_{cont}, Y_{f} ] \text{, and } \Theta = [ \Theta_1^T, \Theta_2^T ]^T. \]

Finally, consider the following control law

\[
\tau = -K_d \dot{S}_{\delta} + Y J_{\varphi}^T q \left[ -\lambda_d = \eta D F \right]
\]

(32)

\[
\dot{\Theta} = -\Gamma Y^T \dot{S}_{\delta}
\]

(33)

\[
\Gamma \in \mathbb{R}^{m, \mathbb{R}}, K_d \in \mathbb{R}^{m, n}, Y > 0, \eta > 0. \]

We now have the following result.

**Theorem 2.** Consider the constrained robot (1) subject to parametric uncertainty on the robot, the camera, tangential contact friction and joint dynamic friction. Assume that initial conditions and desired trajectories belong to \( \Omega \), consider the adaptive visual servoing force-position controller (33)-(34). If \( K_d \) is large enough and a error of initial conditions are small enough, and if \( \gamma \geq \frac{d}{dt} \left\{ R_a \Theta \right\} \left[ S_{\delta} + J R_{\text{Rine}} - J R_{\text{Rine}} \right] \frac{d}{dt} \]

and
\[ \gamma_F \geq \frac{d}{dt} \left[ (J_x J_x^T(q))^{-1} J_x \dot{S}_q \right] \] then exponential convergence of visual and force tracking errors is guaranteed.

**Proof.** With the very same Lyapunov function of theorem 1, we obtain the time derivative, along trajectories of the closed loop of (33)-(34) and (1), as:\[ \dot{V} \leq -\lambda_{\text{min}} (K_d) \| \dot{S}_q \|^2 - \eta \| S_t F \| + \| \dot{S}_q \| \psi - \dot{V}_f, \] where \( \dot{V}_f = \sigma_0 S_q^T [z + \sigma_0 \tanh(\lambda_s S_t)] - \sigma_0 S_q [-z h(x) + \sigma_0 \tanh(\lambda_s S_t)] > 0, \) see (Garcia-Valdovinos and Parra-Vega, 2003). From here on, we proceed exactly as in proof of Theorem 1, details are therefore omitted. QED.

8. SIMULATIONS

Robot parameters are taken from a 2 DOF planar robot, also the camera parameters are from SONY DFWVL500 CCD camera. Robot parameters: Mass(6.2)kg, Length(0.4,0.3)m. Camera: \( \theta(90^\circ), \alpha(77772)\text{pix/m}, z(1.5)m. \) Friction parameters: \( \sigma_0(30000), \sigma_1 = \sigma_2(2), \alpha_0(4), \alpha_1(0.4), \tilde{\gamma}_s(0.01). \) The desired trajectories for the simulation is \( x_s = \alpha R[xcd;ycd] + \beta, xcd = 0.5; ycd = 0.5 + r \sin(\omega \ast t); r = 0.1, \omega = 0.5. \) The contact surface is a plane parallel to plane \( YZ \) and over \( x = 0.5. \)

Feedback gains are \( \Gamma = \text{diag}(1), \kappa_f = 20, \gamma_f = 3.0, \eta = 0.029, \beta = 1.0, K_d = \text{diag}(90), \alpha = \text{diag}(40), \kappa_s = 20, \gamma_s = \text{diag}(7.8). \)

9. CONCLUSIONS

A novel adaptive servo visual scheme for a constrained dynamical robot system is presented. The main feature of this scheme is the ability to fuse image coordinates and contact forces. Local exponential convergence arises for image position-velocity and contact forces even when robot parameters, camera parameters and contact friction are unknown. Additionally, it is proposed a compensator of uncertain joint dynamical friction, which has never treated in the literature of visual servoing, but it is particularly important in contact motion tasks, because the motion regime is slow, with velocity reversals. Simulations confirm the predicted stability properties.

**REFERENCES**


